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Non-Linear Budget Constraints and Consumer Demand: An Application to Public Programs for Residential Housing

BY

JOHN M. QUIGLEY

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Non-linear Budget Constraints and Consumer

Demand: An Application to Public Programs

for Residential Housing\*

bу

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#### I. Introduction

In contrast to many other commodities traded in the economy, the housing services commodity includes a diverse bundle of attributes priced in a complex way. Because of the high costs of transforming housing bundles, arbitrage is generally impossible; thus it is to be expected that the prices of individual attributes vary jointly in a non-linear way, even in market equilibrium. Indeed, in the housing literature there has been a great deal of attention paid empirically to the joint pricing problem, and the literature on hedonic housing prices is voluminous. \( \)

Despite at least three discussions of the analogy between shopping for housing and shopping in a supermarket (c.f., Freeman, 1979; Kain and Quigley, 1975; Triplett, 1975), the correct interpretation of these hedonic functions was widely misunderstood until the work of Sherwin Rosen (1974) appeared. Rosen's analysis indicated rigorously the relationship between the offer functions of suppliers, the bid functions of demanders and the hedonic structure of prices. Empirically, Rosen suggested a procedure for estimating the compensated demands of consumers, a procedure which has been implemented in the housing context in one recent paper (Witte, Sumka, and Erekson, 1979). More recently, it has been shown (Brown and Harvey Rosen, 1980) that the estimation procedure, as originally suggested, contains a fatal flaw and cannot be employed to identify the structural parameters of suppliers' offers or demanders' bids.

This paper applies the hedonic theory to the housing market, and indicates how sufficient structure can be placed on the problem to identify and estimate the compensated demands of consumers. In the empirical section, the methodology is employed to evaluate the net benefits of a somewhat stylized program for housing improvement.

<sup>1.</sup> For example, a recent paper by Freeman (1979) compares the results of 18 hedonic analyses of housing prices which consider air quality as a component of the housing bundle.

Section II below summarizes the hedonic theory of the housing market and investigates where additional structure can be imposed on the problem. In this discussion we simplify the analysis somewhat and ignore the production side, for the most part. We consider the problem of allocating existing housing stocks to housing consumers. Section III indicates how market information and behavioral restrictions can be used to identify and estimate compensated demand curves for housing components. The analysis in Section III is related to the work of Harrison and Rubinfeld (1978), Murray (1975, 1978) and Walters (1975). The empirical analysis is reported in Section IV. For the most part continuity and differentiability are assumed throughout the analysis. In another paper (Quigley, forthcoming), the discrete version of this problem is considered in detail.

#### II. Hedonic Relations and Bid Rents

At any moment, observations in the market provide information on the vector h of housing attributes  $h_1, h_2, \ldots, h_n$  which completely describe the services provided by each unit, and the level of expenditures R, which each unit commands. From observations on the dwellings and their associated housing attributes in a single competitive market,

$$(1) \qquad \qquad R = p(h)$$

describes the relationship between the characteristics of housing services and the rents they command. This hedonic price function indicates the total cost of each collection of attributes. Assuming continuity, the marginal price for any attribute  $\frac{\partial p(h)}{\partial h}$ , is determined, at any level of the other attributes. The hedonic price function has been mentioned in the literature for almost

twenty years (Griliches, 1961) and has been widely used in the development of index numbers (e.g., for automobiles). The notion of analyzing the marginal prices of housing attributes through the computation of hedonic indices was advanced in the late 'sixties (e.g., Ridker and Henning, 1967) and developed extensively in the 1970s.

Consider the supermarket analogy. Clearly the "correctly" specified and estimated hedonic relationship tells us no more about consumers' valuations of individual grocery items than the information that can be obtained by visiting a more conventional grocery store and observing the prices on the shelves. If consumers are competitive, then relative prices reveal marginal rates of substitution (regardless of supplier behavior). If suppliers are competitive, relative prices measure marginal rates of transformation.<sup>2</sup>

<sup>2.</sup> A number of earlier studies used estimated hedonic relationships to make inferences about the level of benefits from public activities, particularly improvements in air quality. The common thrust of these attempts has been to estimate the benefits of a change in, say, air quality from  $h_j$  to  $h_j^2$  at location j from statistical estimation of equation (1). Differentiate equation (1) to find the marginal price of clean air  $\frac{\partial p(h)}{\partial h}$  at location

j, holding all other attributes constant (Polinsky and Rubinfeld, 1977). The aggregation of property values changes across all locations,

 $<sup>\</sup>sum_{j} \left[ \frac{\partial p(h)}{\partial h} \right]_{i} (h_{j}^{*} - h_{j})$ 

<sup>,</sup> represents a prediction of the market's valuation of a specific program, namely the improvement of air quality from h to h\*. Although this procedure has been used extensively to estimate the benefits of public programs (cf., Ridker and Henning, 1967), it is clearly based upon faulty reasoning. In the first place, it need not be true that this procedure estimates the change in market values from any program accurately. In general, the aggregate change in values is misestimated because the property value prediction for each location supposes that everything else (including the air quality at <u>all other</u> locations) is unchanged (Edel, 1971). In the second place, even if this procedure correctly predicts property value changes, this change need not correspond to the benefits (or aggregate willingness to pay) for any public program (see Oron, Pines, and Sheshinski, 1974).

In the absence of additional behavioral assumptions, the "correct" form of the hedonic relationship is largely an empirical issue. There has apparently been only one study (Goodman, 1978) which has attempted rigorously, using Box-Cox techniques, to infer the statistically correct functional form for a hedonic relation for any data set for residential housing.

#### A. The Derivation of the Hedonic Price Function

Housing is a vector of attributes. Without loss of generality, assume that there are two attributes,  $h_1$  and  $h_2$ . Consumers have well defined preferences over housing attributes and other goods x,

(2) 
$$U = u(x, h_1, h_2)$$

$$U_x, U_{h_1}, U_{h_2} > 0;$$

$$U_{xx}, U_{h_1h_1}, U_{h_2h_2} < 0 .$$

Each consumer is endowed with income y which is spent on other goods, with a price of one, and housing attributes. Housing attributes are jointly priced;  $p[h_1, h_2]$  represents the total cost of consuming a dwelling unit with attributes  $h_1, h_2$ . It is the "time" hedonic price relationship in the market. Each consumer chooses one dwelling unit, so the budget constraint can be represented by

(3) 
$$y = x + p[h_1, h_2]$$

Maximizing (2) subject to (3) yields the equilibrium of the individual consumer

$$\frac{U_{h_1}}{U_{x}} = \frac{\partial P}{\partial h_1} \qquad ; \qquad \frac{U_{h_1}}{U_{h_2}} = \frac{\partial P/\partial h_1}{\partial P/\partial h_2}$$

$$\frac{U_{h_2}}{U_x} = \frac{\partial P}{\partial h_2} \qquad ; \qquad y = x + p[h_1, h_2]$$

In general the four equations can be solved for three demand equations the quantities x,  $h_1$ , and  $h_2$  consumed, given income and exogenous prices.

Note two special features of this formulation. First, since housing attributes are jointly priced according to the relation  $R = P[h_1, h_2]$ , no simple inverse exists. Second, the assumption that each consumer chooses one dwelling implies that the utility function can be written as

(5) 
$$U = u(y - p[h_1, h_2], h_1, h_2)$$

Holding constant (but not necessarily measuring) the value of the utility function, we ask the following question: at known income  $y^0$ , what is the maximum amount that can be offered for all other bundles of attributes

leaving the consumer equally well off? The highest bid B is clearly

(6) 
$$u(y^{0} - p[h_{1}^{0}, h_{2}^{0}], h_{1}^{0}, h_{2}^{0}) = U^{0} = u(y^{0} - B, h_{1}, h_{2})$$

Equation (6) defines an implicit relationship between B, bid payments, and housing attributes  $h_1$  and  $h_2$  yielding identical levels of satisfaction. Since the initial endowment of the consumer is arbitrary, the bid relationship varies with income and the level of individual utility. The family of bid relationships can be written more generally as

(7) 
$$B = f(U, y, h_1, h_2)$$

For any endowment, y is fixed at  $y^0$  and (7) traces out a family of curves. Each curve indicates the bid for all combinations of housing attributes at an arbitrary level of utility. The restrictions on the utility function  $U_h > 0$ ,  $U_{hh} < 0$  imply that  $B_h > 0$ ,  $B_{hh} < 0$ , that is, the bid for any housing attribute is increasing at a decreasing rate. In addition, the bid in terms of the numeraire x for a marginal increment of h is, in equilibrium, the marginal rate of substitution between h and x

$$\frac{\partial B}{\partial h_1} = \frac{U_{h_1}}{U_{x}}$$

$$\partial B = \frac{U_{h_2}}{\partial h_2}$$

As noted by Rosen,  $\partial B/\partial h$  is nothing other than the compensated (Hicksian) demand curve for h, that is, it represents the demand price for additional units of h at a constant level of utility.

Two properties about these relations are observed from market data. First households with incomes  $y^0$  are observed to choose  $h_1^0$  and  $h_2^0$  and to bid  $p[h_1^0, h_2^0]$  for a dwelling. Thus

(9) 
$$B = f(U^0, y^0, h_1^0, h_2^0) = p[h_1^0, h_2^0]$$

The solution to Equation (9) provides the common utility index  $U^0$  of all consumers with endowment  $y^0$ . Second, from the first order conditions (4), the derivative of the bid function (9) must equal marginal prices in the market

(10) 
$$\frac{\partial B}{\partial h_{1}} = \left| \frac{\partial f(U^{0}, y^{0}, h_{1}, h_{2}^{0})}{\partial h_{1}} \right| = \left| \frac{\partial P[h_{1}, h_{2}]}{\partial h_{1}} \right| \begin{pmatrix} h_{1}^{0} \\ h_{2}^{0} \end{pmatrix}$$

$$\frac{\partial B}{\partial h_{2}} = \left| \frac{\partial f(U^{0}, y^{0}, h_{1}^{0}, h_{2})}{\partial h_{2}} \right| = \left| \frac{\partial P[h_{1}, h_{2}]}{\partial h_{2}} \right| \\ h_{1}^{0} \\ h_{2}^{0}$$

At the equilibrium chosen by the consumer, the value of the bid rent curve must equal the value of hedonic price relation, and the partial derivative of the bid rent curve must also equal the partial derivative of the hedonic price function. This implies that the bid rent function for each housing attribute must be tangent to the hedonic price relation.

Alternatively, the hedonic price relation must itself be the equation of the envelope of the bid rent functions for each attribute.

#### III. The Imposition of Behavioral Restrictions

As demonstrated by Brown and Harvey Rosen (1980), joint estimation of the set of n+1 equations

(11) 
$$R = f(h_1, \ldots, h_n)$$

$$\frac{\partial R}{\partial h_1} = g_1(h_1, \ldots, h_n, Z_1)$$

$$\frac{\partial R}{\partial h_n} = g_n(h_1, \ldots, h_n, Z_n),$$

where  $Z_1$ , . . . ,  $Z_n$  are exogenous shift variables, cannot possibly provide estimates of the structural parameters identifying the bid functions of consumers. The reason, of course, is that the marginal prices of housing attributes are deterministic functions of the set of attributes; that is, they contain no information beyond that contained in the observed sample of attributes. In this section, we investigate how specific restrictions can be used to identify the functional relationship. Two questions are raised. First, if assumptions are made about the structure of consumer preferences, what restrictions are imposed upon the market-wide hedonic price relationship? Secondly, if any functional form for the hedonic relation is stipulated exogenously, what information does this supply about the underlying utility functions?

The general answer to the first question is quite complicated, but a simple example demonstrates that, in fact, few meaningful restrictions can be imposed.

Consider a very simple model; assume for convenience, that housing is a single valued commodity. We observe the utility maximizing behavior of households as price takers and infer the shape of the hedonic relationship.

Assume the utility function is Cobb-Douglas with

$$U = Ax^{\alpha}h^{\beta}$$

where A is arbitrary and  $\alpha$  and  $\beta$  are known parameters. Given a market price relationship p(h), maximizing (2') subject to (3) yields the market demand curve for housing:

$$h = \frac{\beta}{\alpha} \frac{[y-p]}{p'}$$

At the point chosen by a household of given income

(7') 
$$B(y^{0},U^{0},h) = y^{0} - \left(\frac{U^{0}}{A}\right)^{\alpha}h^{-\alpha}$$

For any income, this bid rent function describes a family of isoutility curves, increasing in h at a decreasing rate. At the point chosen, it must be true that

Figure 1 illustrates the information available for two consumers, with incomes of y=3 and y=12. The figure indicates the family of bid rent curves for the utility function,  $U=Ax^{\frac{1}{2}}h^{\frac{1}{2}}$ . With this information, the exact shapes, though not the locations of the bid rent curves are known. The hedonic function is the envelope of the bids of these consumers. If the hedonic function were known,  $p(h)=h^2$  in figure 1, then we would observe the lower income household's choice of one unit of housing at a price of 1, and

the higher income household's choice of two units of housing at a price of 4. It is easily verified that each choice is the utility maximizing solution to equations (7') and (10') for a consumer with given income.

As shown in equation (4'), the Cobb-Douglas utility function implies that higher income households choose larger amounts of h and pay larger amounts P(h). Thus as the figure is drawn, each point higher up on P(h) corresponds to the tangency of the bid rent function of a consumer with higher income. Consider the function h(y), giving the housing chosen in equilibrium as a function of income. The Cobb-Douglas utility function insures that h'(y) > 0.

Now suppose only the utility functions and endowments are known exactly  $(\underline{i.e.}, \alpha \text{ and } \beta \text{ are known for households receiving exogenous income}$   $(y_1, y_2, \dots, y_n)$ . What restrictions does this information impose on the hedonic structure of market prices?

The consumer's problem is to choose h according to the rule:

(12) 
$$\max_{h} A[y - P(h)]^{\alpha} h^{\beta}$$

The first order condition for a maximum can be expressed as

(13) 
$$hp' = \frac{\beta}{\alpha} [y - P(h)]$$

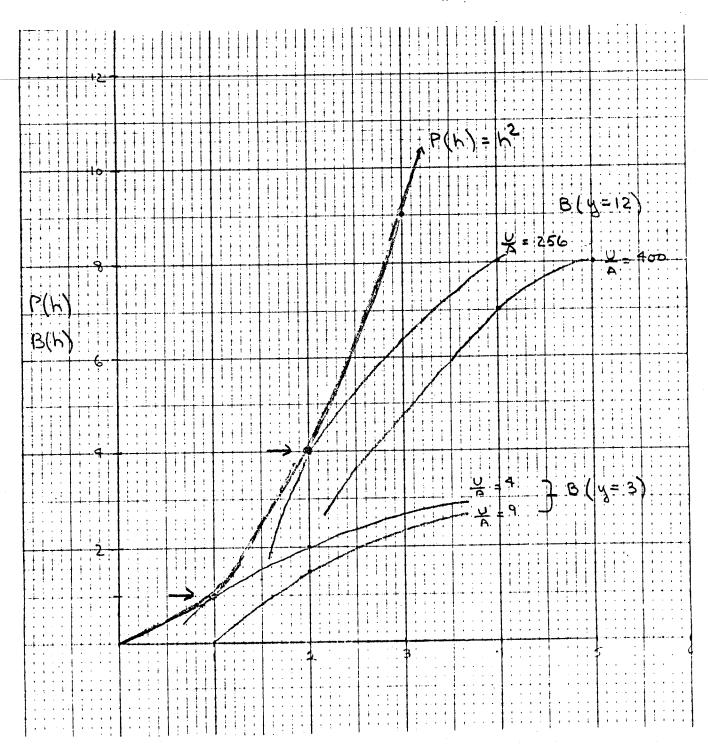
Any strictly concave hedonic function insures that the left hand side of (12) is uniformly increasing in h and the right hand side is uniformly decreasing in h. Thus the solution, the amount of housing chosen by a consumer of any income, is unique. Now consider h(y), an arbitrary monotonically increasing relation between housing and income. Substitute into the first order condition

(14) 
$$h(y)p'[h(y)] = \frac{\beta}{\alpha} \{y - p[h(y)]\}$$

Figure 1

Utility Maximizing Bids for Cobb-Douglas Consumers of Differing Income ( $\alpha = \beta = \frac{1}{2}$ )

Hedonic price:  $P(h) = h^2$ Bid rents:  $B(h) = y - \frac{(U/A)}{h}$ 



and differentiate with respect to y, yielding

(15) 
$$h'(y) = \frac{\beta/\alpha}{h(y)p''[h(y)] + p'[h(y)](\frac{\alpha+\beta}{\alpha})}$$

Housing consumption will increase with income as long as the denominator of the right hand side of (14) is positive. Any concave hedonic function (P'>0; P">0) again insures this result. If individuals act as price takers, the maximization of individual utility is consistent with any hedonic price function which is concave, even in the restricted world of Cobb-Douglas utility functions. The imposition of a quite specialized form on consumer preferences generates only weak restrictions on the form of the hedonic structure of market prices.  $^3$ 

The investigation of this simple and specific model suggests that assumptions about the form of the household utility function imply practically no meaningful restrictions on the shape of the hedonic price surface, in a world where market prices are purely demand determined.

If symmetrical information is known about the suppliers as well as demanders of housing, it may be possible to deduce the exact shape of the hedonic function. Suppose, for example, housing is produced from two factors: w, purchased at constant price  $P_w$ ; and v, at a price of one. Assume the level of V fixed in the short run for each supplier. Now if the production function

$$P'' > \frac{-P'}{h} \left[ \frac{\alpha + \beta}{\alpha} \right]$$

<sup>3.</sup> The necessary condition is, of course weaker. The hedonic function need not even be concave; the denominator of (14) must merely be positive. Know-ledge of the Cobb-Douglas parameters imposes only the following condition on the second derivative:

for housing is also Cobb Douglas with known parameters

(16) 
$$K v^A w^B = h$$
,

then the offer function for a firm, holding its profits and fixed factors constant at  $\Pi_{_{\rm O}}$  and  $v_{_{\rm O}},$  will be of the form:

(17) 
$$O(h) = \Pi_O + V_O + P_W \left(\frac{h}{KV_O}\right)^{\frac{1}{B}}$$

where  $\Theta$  (h) is the price at which suppliers offer h for sale. Exact knowledge of the parameters of the production function and the endowments ( $v_o$ ) of suppliers, may be combined with symmetrical knowledge about demanders to identify points on the hedonic function.<sup>4</sup>

Now consider the converse problem; assume the hedonic function is known with certainty. What information is generated about the utility function? Presume that the hedonic relation is given exogenously, or else that it is derived by some "best fit" statistical criterion, such as Box-Cox, prior to the analysis.

As is illustrated in figure 2 for a homothetic utility function, if the form of the hedonic function is known and if it is non-linear, then the exact shape of the utility contour can be inferred. Figure 2 illustrates the utility contours of an individual and the non-linear budget constraint

<sup>4.</sup> For example, in figure 1, the offer curve of a firm will be tangent to the bid rent curve of the consumer with income y=2 and will be tangent to the hedonic function if  $\Theta$  (h = 2) = 4 and  $\frac{\partial\Theta}{\partial h}$  (h = 2) = 4. If it were known that  $B = \frac{1}{2}$  and that  $P_{W} \left[\frac{1}{Kv_{O}}A\right]^{2} = 1$ , then the bid and offer curves would be tangent at the point P = 4, h = 2.

implied by market price function. With a linear budget constraint, observations on identical consumers of different incomes lie along a single ray, say ray I from the origin. With homothetic preferences, the slopes of the three indifference curves along the ray are identical at A, B, and C. Thus, with linear prices, the estimation of utility contours (as distinct from income expansion paths) is impossible.

With a non-linear budget constraint, however, the slopes of the indifference curves at B' and C' differ from each other and from the slope at A'. If preferences are homothetic, however, the slope at B" along ray II must equal the slope at B' and the slope at C'.

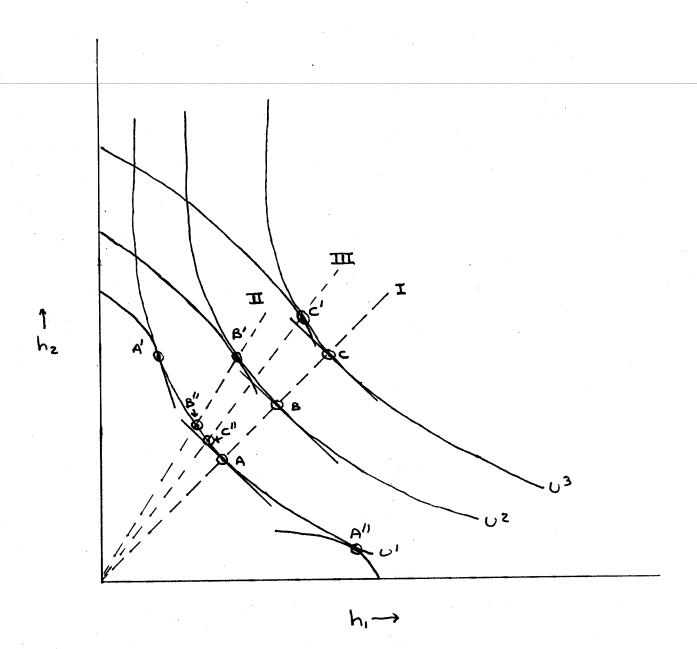
Observations on identical individuals of different incomes are sufficient to trace out the shape of any indifference contour if the budget constraint is non-linear and if preferences are homothetic; the family of contours can always be approximated numerically for any sample of observations on households.

Moreover, with nonlinear prices and several housing attributes, the trace of the budget constraint in the two dimensional plane may vary for households of the same incomes and preferences. Two households of type U' may elect differing budget constraints and different bundles A' and A" of  $h_1$  and  $h_2$  simply by their choice of  $h_3$ .

Thus, it should be clear that the assumption of homotheticity of preferences is not even necessary if utility contours are to be traced out numerically. For estimation of the utility contours by statistical means, it is only necessary that each point B' on  $U^2$  be associated systemically with some point B" on  $U^1$ . Any assumption about the functional form of the utility function permits the parameters defining the contours to be estimated.

Figure 2

Calculation of Utility Contour for Homothetic Preferences with Non-Linear Budget Constraint



Suppose for example there are n housing components and the utility function is GCES (See Murray 1975; or especially Murray 1978).

(18) 
$$U = \left(\sum_{i=1}^{n} \alpha_{i} h_{i}^{\beta_{i}} + x^{\epsilon}\right)^{\phi}$$

where  $\phi$  is arbitrary. In general, unless  $\beta_i = \beta_j = \epsilon$  the function depicts non-homethetic preferences. Maximizing (18) subject to (3) yields, n first order conditions of the form

(19) 
$$\log \frac{\alpha_i \beta_i}{\varepsilon} + (\beta_i - 1) \log h_i - (\varepsilon - 1) \log x = \log \frac{\partial p}{\partial h_i}$$

and the budget constraint. The system includes n+l commodities, n+l equations with incomes and the hedonic price function as exogenous. Instrumental estimation of n equations, subject to one cross equation constraint, yields 2n+1 parameters which can be solved for: the  $\alpha$ 's,  $\beta$ 's and the value of  $\epsilon$ . Just as knowledge of the form of the demand curve permits Marshallian demands to be estimated when prices are linear, so knowledge of the form of the utility function permits the Hicksian demands to be estimated when prices are non-linear.

There appear to be a few papers which have utilized related techniques to estimate household willingness to pay for jointly priced commodities or to perform cost-benefit calculations. A book by Walters (1975) estimates a homothetic (Cobb-Douglas) utility function for "quiet" and other goods; a paper by Murray (1975) relies on a more complex analog to this procedure to estimate the benefits of public housing; a more recent methodological discussion (Murray, 1978) explicitly considers the estimation of GCLS utility parameters using budget shares derived from hedonic prices; finally, a recent paper by Harrison and Rubinfeld (1978) relies upon an <u>ad hoc</u>, but related, methodology to estimate household willingness to pay for clean air.

#### IV Empirical Analysis

In this section, we apply the methodology of Section III to analyze household benefits of (a highly stylized version of) a public housing investment In 1976 a sample of households in Santa Ana, El Salvador was selected to receive housing subsidies under a "sites and services program." Under this program (or rather the highly stylized version of the program considered here), households were offered the opportunity to consume a specified collection of housing attributes at a predetermined price. Available for analysis are two observations on each of a sample of low income dwelling units rented in the private economy in Santa Ana. Associated with each dwelling unit is a set of variables describing its characteristics, as well as information on the socio-demographic characteristics of the particular household renting it. The first observation, taken in 1976, includes a subset of households who had been selected to receive public subsidy, but who were at that time living in the private economy. Also included is a sample of nonparticipant households in the private economy. At the date of the second observation, 1979, the households selected to participate had recently moved to their subsidized dwelling units. The data are from a combination of two sampling frames. For non-participants, observations represent a longitudinal sample of dwelling units, not necessarily of households; for participants, they are observations on the same households before and after the subsidy was received.

Table 1 indicates the data available on the physical attributes of rental dwelling units and their prices in 1976 for a combined sample of 253 dwellings. Individual dwellings are located in three types of settlement:

tenements (mesones); illegal subdivisions (colonias illegales); or shanties (turgurios). Measures of physical size of dwellings as well as estimates of the lot size associated with each unit are available for both 1976 and 1979. For tenement dwellings the lot size is recorded as zero. Information is also available on construction materials for selected components of dwellings and for the kinds of sanitary services and amenities provided. Overall quality assessments of aspects of the dwellings (not shown) were gathered only in 1979, and the length of tenure of residents is only known in 1979. Unfortunately, no information is available on the characteristics of the micro-environments or neighborhoods in which these dwellings are located.

The attributes of the 253 dwelling units available for analysis for 1976 include three cardinal measures:  $x_1$ ,  $x_2$ , and  $x_3$  (lot size; roofed area or living area; and the number of rooms), and two binary variables:  $x_4$  and  $x_5$  (signifying the presence of electricity and running water).

In addition, 8 dummy variables report the attributes of plumbing facilities, and 3 sets of 6 dummy variables report the condition of the roof, walls, and floors respectively.

With the exception of the three size variables, all other attributes of dwelling units are categorical in nature, that is, they can be represented in regression analyses by a series of dummy variables, but there is no convenient cardinal metric.

As discussed in the introduction, neither the correct set of independent variables nor the correct function form is known <u>ex ante</u>. The analysis proceeds by investigating both issues simultaneously. The analysis which justifies the combination of the dummy variables reporting condition into

Table 1

## Means and Standard Deviations of Housing Variables

#### 253 Observations--1976 Data

Variable	Mean	Standard Deviation	Minimum	Maximum
Number of rooms: X <sub>1</sub>	1.300	0.515	1.0	3.0
Living area: X <sub>2</sub> (meters <sup>2</sup> )	33.411	19.555	10.0	99.0
Lot size: X <sub>3</sub> (meters <sup>2</sup> ) Entire sample Mesones (225 obs.) Non-mesones (28 obs.)	2.260 0.0 20.42	8.619 -	0.0	99.0 0.0
<pre>Electricity: X<sub>4</sub>   (1 = available)</pre>	0.893	-	0.0	1.0
Piped water: X <sub>5</sub> (1 = available)	0.917	-	0.0	1.0
Sanitary quality:	1.881	0.684	1.0	, 4.0
(1 to 4) Roof condition:	2.000	0.126	1.0	3.0
(1 to 3) Floor condition:	1.932	0.250	1.0	3.0
(1 to 3) Wall condition:	2.036	0.348	1.0	3.0
<pre>(1 to 3) Aggregate condition:</pre>	5.968	0.469	3.0	7.0
(1 to 9) Rent (colones per month)	22.636	12.176	8.0	80.0

the few quality indices reported in table 1 is relegated to appendix 1.

#### A. The Hedonic Price Relationship

Hedonic regressions relating monthly rent to the characteristics of the housing bundles are reported in appendix 2 for four common specifications based upon 253 dwelling units observed in 1976. A comparison of parameters and explained variance yields only a weak preference for the semi-log specification. In this section, we apply the method first suggested by Box and Cox (1964) to the problem.

Define the following family of transformations of the dependent variable rent (R):

(20) 
$$R^{(\lambda)} = (R^{\lambda} - 1)/\lambda \qquad \text{for } \lambda \neq 0$$
$$= \log R \qquad \text{for } \lambda = 0$$

This family of transformations, generated by  $\lambda$ , is well defined for all R > 0. If it is assumed that for the regression equation,

(21) 
$$\operatorname{rent}^{(\lambda)} = b_0 + \sum b_i X_i + \dots + u ,$$

the u's are normally distributed with zero mean and variance  $\sigma^2$ , we may estimate b,  $\lambda$  and  $\sigma$  by standard maximum likelihood techniques. In addition, as has recently been shown by Olsen [forthcoming], if normality of the u's is inappropriate, this can be taken into account explicitly in the estimation procedure. Since the dependent variable, rent, with a mean of 22.6 and a standard deviation of 12.2, is truncated at zero, some departure from normality in u is indicated.

Table 2 presents the values of the log likelihood function, obtained by steepest-descent methods, at alternative values of  $\lambda$ .

The first column indicates the results of the "standard" Box-Cox maximization, assuming normality of errors. The second column indicates the results of the more general estimation, due to Olsen, which takes into account the truncation of the dependent variable.

In either case, the value of the likelihood function is maximized at  $\lambda$  = -0.4. One difference between the two procedures is the flatness of the likelihood function. When non-normality is appropriately taken into account, the semi-log function ( $\lambda$  = 0) reported in appendix 2 is within the 95 percent confidence interval for  $\lambda$  as indicated by the likelihood ratio test (2 x [861.95 - 860.12] = 3.66  $^{\sim}$   $\chi^2$  with 1 degree of freedom). This is not the case when the "standard" Box-Cox procedure is applied ( $\chi^2$  = 10.68).

Table 3 reports to coefficients of the Box-Cox specification at the maximum likelihood. The evidence presented in appendix lindicates that the condition of dwelling units can be represented by a single cardinal index, and the sanitary quality by another cardinal index. Six of the seven variables have highly significant coefficients; the tratio for the lot size variable is 1.27.

The hedonic results reported in table 3 and in the appendix permit the computation of the marginal prices for housing attributes facing each

Table 2

### Values of Log Likelihood Function for Box-Cox Transformation

, , <u><b>λ</b></u>	Assuming Normality a	Non-normality <sup>b</sup>
0.2	-872.68	-864.09
0.0	-865.48	-861.95
-0.2	-864.09	-861.35
-0.4	-860.14	-860.12
-0.6	-861.69	-861.67
-0.8	-865.80	-865.01

#### Notes:

a. The likelihood function is defined as

$$L^{1}(y|b, x, \sigma, \lambda) = J(\lambda; y) \prod_{i=1}^{n} \frac{1}{\sigma} f\left(\frac{y_{i}^{(\lambda)} - \sum_{j} b_{j} x_{ij}}{\sigma}\right)$$

where 
$$J(\lambda; y) = \prod_{i=1}^{n} \left| \frac{d y_i^{(\lambda)}}{d y_i} \right| ; y_i > 0$$
  
=  $(\lambda - 1) \sum_{i=1}^{n} \log(y_i)$ 

and  $f(\cdot) = \text{standard normal p.d.f.}$ 

b. The amended likelihood function is defined as

$$L^{2}(y|b, x, \sigma, \lambda) = J(\lambda; y) \prod_{i=1}^{n} \frac{1}{\sigma} f(\frac{y_{i}^{(\lambda)} - \sum_{j} b_{j} x_{ij}}{\sigma^{j}}) / P_{i}$$
where
$$P_{i} = 1 - F[(-1/\lambda - \sum_{j} b_{j} x_{ij})/\sigma] \qquad ; \quad \lambda > 0$$

$$= 1 \qquad \qquad ; \quad \lambda = 0$$

 $= F[(-1/\lambda - \sum_{j} b_{j} x_{ij})/\sigma] \qquad ; \quad \lambda < 0$ 

and  $F(\cdot) = \text{standard normal c.d.f.}$ 

Table 3

## Hedonic Regressions Computed from Box-Cox Transformation of Dependent Variable and Market Wide Marginal Prices

#### 253 Observations--1976 Data

<u>Variable</u>	Coefficients	Mean marginal prices	
Number of rooms	0.104 (7.32) <sup>a</sup>	8.770 (3.72)b	
Living area (meters <sup>2</sup> x 1000)	0.776 (2.14)	6.560 (5.30)	
Lot size (meters <sup>2</sup> x 1000)	1.050 (1.27)	8.870 (7.20)	
Electricity (1 = available)	0.048 (2.06)	4.081 (3.29)	
Piped water (1 = available)	0.059 (2.25)	2.816 (1.01)	
Sanitary quality	0.235 (2.18)	1.988 (1.62)	
Aggregate condition	0.029 (1.88)	2.434 (1.99)	
Intercept	1.266 (13.68)		
$R^2$	0.342		
R <sup>2</sup> in original space	0.490		
SEE/mean	0.063		

Note a: t-ratios in parentheses b: standard deviations in parentheses

household. Table 3 also presents the mean marginal price for each attribute facing households in the sample computed from the Box-Cox hedonic regression. For example, an additional room costs on average 8.77 colones per month but the cost of an additional room varies considerably; the standard deviation is 3.72.

B. The Estimation of Household Preference Functions

For any of the four non-linear forms of the hedonic price function, the marginal price for each attribute facing each household can be computed, for those housing attributes which are continuous. In this section we use the Box-Cox marginal prices summarized in table 3 to estimate the parameters of the generalized constant elasticity of substitution (GCES) utility function. <sup>5</sup>

The hedonic results indicate that seven characteristics of housing command prices in the Santa Ana housing market. Five of these are measured by continuous variables: rooms, living area, lot size, condition, and sanitary quality. Two are discrete variables: the presence of piped water and electricity.

Assume a utility function for households of the following (GCES) form:

$$U = \begin{bmatrix} 5 & \alpha_i h_i^{\beta_i} + \alpha_6 h_6 + \alpha_7 h_7 + x^{\epsilon} \end{bmatrix}^{\phi}$$

where

 $h_1 = rooms/person$ 

h<sub>2</sub> = living area/person

<sup>5.</sup> Estimates based on the hedonic regressions reported in the appendix are available on request.

 $h_3$  = lot size/person

 $h_4 = condition$ 

 $h_5$  = sanitary quality

 $h_6$  = piped water

h<sub>7</sub> = electricity

x = other goods

Equation (22) specifies that households prefer more space per person, as measured by rooms, living area and lot size; the remaining characteristics are not expressed in per capita terms.  $\phi$  is arbitrary and the remaining 13 parameters define household preferences.

As discussed above, maximization of (22) subject to the budget constraint yields 5 equalities (which contain only continuous variables) of the form:

(23) 
$$\log \frac{\alpha_i \beta_i}{\epsilon} + (\beta_i - 1) \log h_i - (\epsilon - 1) \log x = \log \frac{\partial p}{\partial h_i}$$

as well as two inequality conditions.

Estimation of the system of five equations represented by (23) yields 11 parameters, which can be solved for 11 of the parameters of the utility function:  $\alpha_1$  through  $\alpha_5$ ,  $\beta_1$  through  $\beta_5$  and  $\epsilon$ .

In the special case where  $\beta_j$  =  $\beta_j$  =  $\epsilon$  =  $\delta$  the system of equations reduces to

(24) 
$$\log \alpha_i + (\delta - 1) \log \left(\frac{h_i}{x}\right) = \log \frac{\partial P}{\partial h_i}$$

Table 4 presents the results of instrumental estimation of these systems of equations for the sample of households in the private housing market in Santa Ana in 1977. The sample includes 249 households for which good data on incomes and family sizes were available as well as data on housing characteristics and monthly rents. The table presents estimates of the 11 parameters of the GCES function and the 6 parameters of the CES specification.

In each case the dependent variable is the marginal attribute price computed from the Box-Cox hedonic function.

The results reported in table 4 also provide a test of the hypothesis that the utility function is CES rather than GCES. The F-ratios indicate that the hypothesis is rejected by a wide margin.

Table 4

#### Estimates of Utility Function Parameters

Parameter		GCES		CES
$\log \frac{\alpha_1^{\beta_1}}{\varepsilon}$		1.744 (11.14) <sup>a</sup>		
log α <sub>l</sub>				1.852 (15.30) <sup>a</sup>
$\log \frac{\alpha_2^{\beta_2}}{\epsilon}$		-3.526 (19.30)		
log ∞ <sub>2</sub>				-3.002 (42.94)
$\log \frac{\alpha_3^{\beta_3}}{\epsilon}$		-3.067 (21.11)		
log α <sub>3</sub>				-2.740 (23.05)
$\log \frac{\alpha_4^{\beta_4}}{\varepsilon}$		-2.517 (3.07)		
log α <sub>4</sub>			i k	0.611 (8.67)
$\log \frac{\alpha_5^{\beta_5}}{\varepsilon}$		-0.242 (1.63)		
$\log \alpha_5$				0.393 (4.40)
(β <sub>1</sub> - 1)	+	0.199 (3.04)		
(β <sub>2</sub> - 1)		0.059 (1.05)		
(β <sub>3</sub> - 1)		0.041 (1.03)		
(β <sub>4</sub> - 1)		1.521 (3.33)		
(β <sub>5</sub> - 1)		0.415 (3.84)		
-(ε - 1)		0.090 (3.39)		
(δ - 1)				-0.013 (1.72)
R <sup>2</sup>		0.909		0.902
F-ratio <sup>b</sup>		19.00		

a. t-ratios in parentheses b. test of the hypothesis that  $\beta_1$  =  $\beta_2$  = . . .  $\beta_5$  =  $\epsilon$  =  $\delta$ 

#### C. Exercising the Model

In this section we use the utility function estimates derived from the Box-Cox prices to impute the private benefits of the stylized housing program.

Two of the housing attributes which appear in the utility function—the presence of piped water and electricity—are inherently discrete, and thus do not appear in the equations describing the marginal conditions of consumption.

A methodology for estimating the parameters,  $\alpha_6$  and  $\alpha_7$ , associated with these attributes is reported in appendix 3. Combining the results reported there with those in table 4 yields the following representations of household utility 6 (25) GCES

```
U = [2.349 (piped water) + 2.864 (electricity) + 5.354 (rooms/person) ^{1.199} + 0.025 (area/person) ^{1.059} + 0.041 (lot/person) ^{1.041} + 0.029 (condition) ^{2.521} + 0.505 (sanitary) ^{1.415} + (other goods) ^{0.910}] ^{\phi}
```

CE\$

U = [2.046 (piped water) + 2.617 (electricity)

+ 
$$6.373$$
(rooms/person)<sup>0.987</sup> +  $0.050$ (area/person)<sup>0.987</sup> +  $0.065$ (lot/person)<sup>0.987</sup> +  $1.842$ (condition)<sup>0.987</sup> +  $1.482$ (condition)<sup>0.987</sup> +  $1.482$ (condition)<sup>0.987</sup> +  $1.482$ (condition)<sup>0.987</sup>

+ 1.842(condition) $^{0.987}$  + 1.481(sanitary) $^{0.987}$  + (other goods) $^{0.987}$   $\phi$ 

where  $\phi$  is arbitrary.

<sup>6.</sup> Although the CES function is rejected in favor of the GCES function by a standard F-test, we report net benefits based upon this function for comparison.

Equation (25) is derived from observing the behavior of a cross section of 249 low income households renting in the private market in 1976. In 1979, 157 of these households were participants in the public program, providing increased levels of housing consumption at subsidized prices.

These expressions can be used to estimate the compensating variation and the income equivalent of the housing program for each participating household. As is well known, these measures of private benefit can be aggregated to provide the total benefit of the program under either of two conditions:

If the urban area is "open," in the sense that utility equalizing migration is instantaneous; or

If the program is "small," in the sense that the presence of the program does not alter the housing market and the structure of relative prices for non-participants.

The existence of a second cross-section of dwellings rented on the private market in 1979, after participants had moved into their subsidized units, permits a test of these conditions.

Table 5 utilizes the 1976 and 1979 cross sections of dwelling units in the private market to test for alterations in the structure of housing prices, which are at least potentially attributable to the subsidy program. For each of the five specifications of the hedonic functions reported in the text and in the appendices, Table 5 reports regression results obtained for the pooled 1976 and 1979 sample and from each sub-sample. For each specification, the t-ratio tests for differences in intercept between 1976 and 1979; the F-ratio tests the null hypothesis that the coefficients of

Table 5

Covariance Tests for Homogeneity of Hedonic Coefficients for 1976 and 1979 Samples of Dwelling Units

	Pooled Sample	1976 <u>Sample</u>	1979 <u>Sample</u>
		Linear	
$R^2$	0.387	0.363	0.399
t	0.249	<del>.</del>	
F	0.873		
		Box-Cox	
$R^2$	0.373	0.343	0.343
t	0.187		
F	0.882		
		Semi Log	
R <sup>2</sup>	0.385	0.357	0.365
t	0.178		
F	0.814		
		Log Log	
$R^2$	0.377	0.345	0.370
t	0.683		
F	0.900		
1.00	the second	Inverse Semi Log	
$R^2$	0.357	0.333	0.394
t	0.714		
F	1.419		•
		•	

the seven housing attributes are identical in 1976 and 1979. The t-ratios and F-ratios are all insignificant by a wide margin, indicating that the structure of housing prices is identical before and after the subsidy program was undertaken.

These results indicate that the program is sufficiently "small," or that the urban economy is sufficiently "open," so that all benefits accrue to program participants.

A comparison of the housing conditions enjoyed by the 157 participating households indicate that they received much better housing conditions in 1979 but also paid much higher rents.

Given the smallness of the public program, equation (25) can be used to estimate the equivalent and compensating variation of the program for any household and also to calculate net program benefits. Let  $(h_1, h_2, \ldots, h_7)$  be the initial level of housing consumption at price p, leaving income minus housing price, x, for consumption of other goods; let  $(h_1^*, h_2^*, \ldots, h_7^*)$  be the housing components offered by the program at price p\*, leaving x\* for consumption of other goods.

The equivalent variation of the program for any participant is clearly

(26) 
$$\Delta_1 = \begin{bmatrix} 7 \\ \Sigma \\ i=1 \end{bmatrix} \alpha_i \left( h_i^*^{\beta_i} - h_i^{\beta_i} \right) + x^{*\varepsilon} ]^{1/\varepsilon} - x$$

Similarly the compensating variation is

(27) 
$$\Delta_2 = -\left[\sum_{i=1}^{7} \alpha_i \left(h_i^{\beta i} - h_i^{\beta i}\right) + x^{\epsilon}\right]^{1/\epsilon} + x^*$$

Table 6 presents summary data on household valuations of the amenities provided by the housing program. Despite the high monthly payments charged by the program, the participating households are, on average, considerably better off on their own terms. The average amount of money which could have been given to households in 1977 in lieu of the sites and services program is estimated to be 4.4 colones per month or about 2 percent of average income in 1977. The average amount of money which could be subtracted from the income of participating households in 1979, to leave them as well off as they were in 1977 is 5.3 colones per month, or about 2 1/2 percent of average income. It should be noted that the simple correlations among these measures of benefits are larger than 0.95 for the 157 households in the sample. Under the assumptions of the model, the program appears to generate substantial benefits. Capitalized at 10 percent, the average benefit per household is estimated to be on the order of 530-640 colones.

#### V. Conclusions

The conceptual discussion suggests that in the context of urban housing markets it is possible to estimate compensated demand curves and the parameters of ordinal utility functions by utilizing information on the joint pricing of housing attributes.

TABLE 6

# Average Private Valuation of Housing Subsidy Benefits<sup>a</sup>

	GCES utility fund	tion	CES utility function			
	Mean value colones/month	Standard deviation	Mean value colones/month	Standard deviation		
Equivalent Variation	4.389	10.571	4.413	9.812		
Compensating Variation	5.344	10.721	5.280	10.314		

a. Based upon 157 recipient households

The principal simplification arises from the imposition of any functional form for household preferences. If the utility function is parameterized in terms of housing and other goods, an exogenous hedonic price function, giving the locus of tangencies between bid rents and marginal prices, permits estimation of the parameters of the function. In contrast to the estimation strategy originally proposed by Sherwin Rosen, this strategy introduces sufficient structure to identify the compensated demand curves of consumers.

The empirical analysis indicates the estimation of compensated demand curves for housing components, and presents estimates of willingness to pay for a highly stylized version of an urban investment program.

The analysis is, however, limited by the assumption that, for the most part, housing attributes are continuously measured along some cardinal scale. In the empirical analysis presented, this is hardly a major problem, but in some circumstances the assumption of continuity would more severely limit the analysis.

#### Appendix 1

As noted in the text, the measure of sanitary quality consists of 8 dummy variables indicating the presence of particular plumbing and sanitary facilities. Prior to the anlysis, these eight dummy variables (measure I) were aggregated to a single index (measure II) with values range from 1 to 4. For any functional form for the hedonic index, the aggregation of these variables into a single measure may be tested by an F-ratio.

Measuring the condition of the dwelling unit are 6 dummy variables describing the condition of the walls, and 6 more describing the condition of floors and roofs. Measure I consists of this set of 18 dummy variables. Measure II consists of a prior aggregation of these into 3 indices with values from 1 to 3. Measure III consists of each of these indices multiplied by its appropriate area: roofed area, floor area, and wall perimeter. Measure IV consists of the simple sum of the three indices (measure II).

Appendix table A1 indicates the appropriate F-tests for the restricted versus unrestricted regressions for these measures for each of the functional forms of the hedonic regression reported in the text and in appendix 2. The table indicates that the hypothesis that sanitary quality measured appropriately by a single index cannot be rejected. The table also indicates that the hypothesis that dwelling unit condition is measured by a single index cannot be rejected. Appendix table A2 provides more detail on the aggregation of the three indices measuring the quality of floors, walls and roofs.

Table 1A

# F-ratios for Alternative Condition and Sanitary Quality Measures for Four Specifications of Hedonic Index<sup>a</sup>

## 253 Observations--1976 Data

	Con	dition Mea	sure	Sanitary
Restrictive Measure vs Unrestrictive Measure	II vs <u>I</u>	III vs <u>II</u>	IV vs <u>I</u>	II vs <u>I</u>
Specification				
Linear form	.812	.144	.483	.361
Semi log	.403	.199	.481	. 307
Log-log	.703	.173	.174	.406
Inverse semi log	.604	.078	.165	.118
Box-Cox	. 802	.198	.117	.364

## Appendix 2

Hedonic regressions relating housing prices to housing characteristics are reported for the Box-Cox functional form in the text. Appendix table A2 presents similar regressions for the linear form as well as the semi-log, log-log, and inverse semi-logarithmic form. It also presents coefficient estimates for the restricted (measure IV) and unrestricted (measure II) representations of dwelling quality.

Table A2

Alternative Functional Forms and Constraints for Hedonic Regressions

## 253 Observations--1976 Data

	ı		1	. 1	, 		1 <u>.</u>		
Coefficients	Linear 1	Form 2	Semi 3	Log <u>4</u>	Log- <u>5</u>	Log <u>6</u>	Inverse 7	Semi Log <u>8</u>	
Number of Rooms	10.770 (8.38)	10.750 (8.41)	0.374 (7.72)	0.373 (7.71)	0,573 (7.61)	0.574 (7.67)		16.041 (7.98)	
Living Area (meters <sup>2</sup> x 10)	0.661 (2.01)	0.653 (2.00)	0.028 (2.22)	0.028 (2.19)	0.027 (2.16)	0.027 (2.19)		0.068 (2.03)	/Ling
Lot Size (meters <sup>2</sup> x 10)	0.528 (0.70)	0.477 (0.64)	0.034 (1.20)	0,033 (1.15)	0.412 (1.59)	0.405 (1.57)		0.748 (1.08)	
<pre>Electricity   (1 = available)</pre>	2.904 (1.34)	2.823 (1.34)	0.159 (1.95)	0.153 (1.91)	0.154 (1.87)	0.147 (1.82)		2.742 (1.27)	
Piped Water (l = available)	2.759 (1.14)	2.669 (1.12)	0.184 (2.02)	0.178 (1.97)	0.173 (1.84)	0.167 (1.80)		2.373 (0.95)	
Sanitary Quality	3.465 (3.52)	3.501 (3.59)	0.093 (2.51)	0.095 (2.58)	0.178 (2.50)	0.181 (2.55)		6.209 (3.26)	
Floor Condition	1.365 (0.52)		0.077 (0.78)		0.067 (0.47)	,	0.898 (0.23)		
Wall Condition	0.753 (0.39)		0.075 (1.04)		0.142 (1.04)		1.925 (0.53)		
Roof Condition	3.875 (0.75)		0,205 (1,05)		0.258 (0.75)		4,586 (0,50)		
Aggregate Condition		1.233 (0.89)		0.088 (1.68)		0.423 (1.48)		5.917 (0.77)	
Intercept	-17.270 (1.43)	-12.550 (1.51)	1.224 (2.68)	1.419 (4.52)	2.070 (7.44)	1.646 (3.23)		-1.788 (0.13)	
$R^2$	0.363	0.363	0.358	0.357	0.345	0.345	0.333	0.333	
R <sup>2</sup> in Original Space	0,363	0,363	0.384	0.384	0.353	0,353	0.333	0.333	
SEE/mean	0.438	0.436	0.124	0.124	0.125	0,125	0.448	0.446	
F-test of Quality Coefficient Equality		0.144		0.199		0.173		0.078	

#### Appendix 3

Two of the attributes of housing, piped water and electricity, are inherently discrete and their parameters cannot be estimated from the first order conditions—the eleven parameters of the GCES utility function reported in the text are incomplete. They neglect the possibility that households trade off more space for piped water, for example.

In the sample of 249 Santa Ana households, 92 percent have piped water in their dwelling units and 89 percent have electricty. How do households substitute between these two attributes and the other characteristics of the bundle of housing and other goods?

It should be clear that if households are informed customers, then, in equilibrium, all households of the same income and family size will achieve the same level of satisfaction regardless of their consumption choices. Thus, if we consider households of the <u>same</u> income and family size, the computed values of the utility function must be identical. Therefore, within any group of homogeneous households, differences in the computed value of the utility index must be attributable to variations in these amenities.

If we partition households of the same income and family size, we can compute the utility index for four groups of households: Those with both water and electricity  $(U_{11})$ ; those with water  $(U_{10})$ ; those with electricity  $(U_{01})$ ; and those with neither  $(U_{00})$ . For each group of identical households, it must be true that:

$$U_{11} = U_{00} + \alpha_6 + \alpha_7$$
 $U_{11} = U_{10} + \alpha_6$ 
 $U_{11} = U_{01} + \alpha_7$ 

The solution of two of the three equations will provide the values of  $\alpha_6$  and  $\alpha_7$  for each class of identical consumers. This technique does not compare utilities across incomes and family sizes; since the system is overdetermined, however, some averaging within household classes is appropriate.

Competition insures that the utility level of households of the same income and family size is identical. Thus

Consider those 207 households choosing both piped water and electricity. A regression of the computed value of utility  $U_{11}$  from the GCES function reported in the text upon income and family size yields

$$U_{11} = U_{00} + \alpha_6 + \alpha_7 = 4.550 + 5.484 \text{ (income)} - 0.514 \text{ (family size)}$$
  
 $(3.90) + (127.40) + (2.36)$ 

where t ratios are in parentheses.

For each of the 249 households in the sample, the coefficients of equation (12) provide a value of  $\hat{U}_{11}$  or  $\hat{U}_{00}$  +  $\alpha_6$  +  $\alpha_7$ , the utility achieved by households if they had chosen both piped water and electricity. Since utilities must be equalized regardless of whether these attributes are chosen, the computed and estimated values of utility can only differ because of the presence of water or electricity, i.e.,

$$U + \alpha_6$$
 (piped water)  $+ \alpha_7$  (electricity) =  $\hat{U}_{11}$ 

A regression of the difference between the utility level computed from the GCES expression and the utility level computed from the above equation if households had chosen water and electricity upon these dummy variables yields, for the sample of 249 observations:

$$U - \hat{U} = 5.249 - 2.349$$
 (piped water) - 2.864 (electricity)  
(3.107) (1.54) (2.10)

where t-ratios are in parentheses.

Application of the same methodology to the CES utility function yields coefficients of 2.046 and 2.617 for piped water and electricity.

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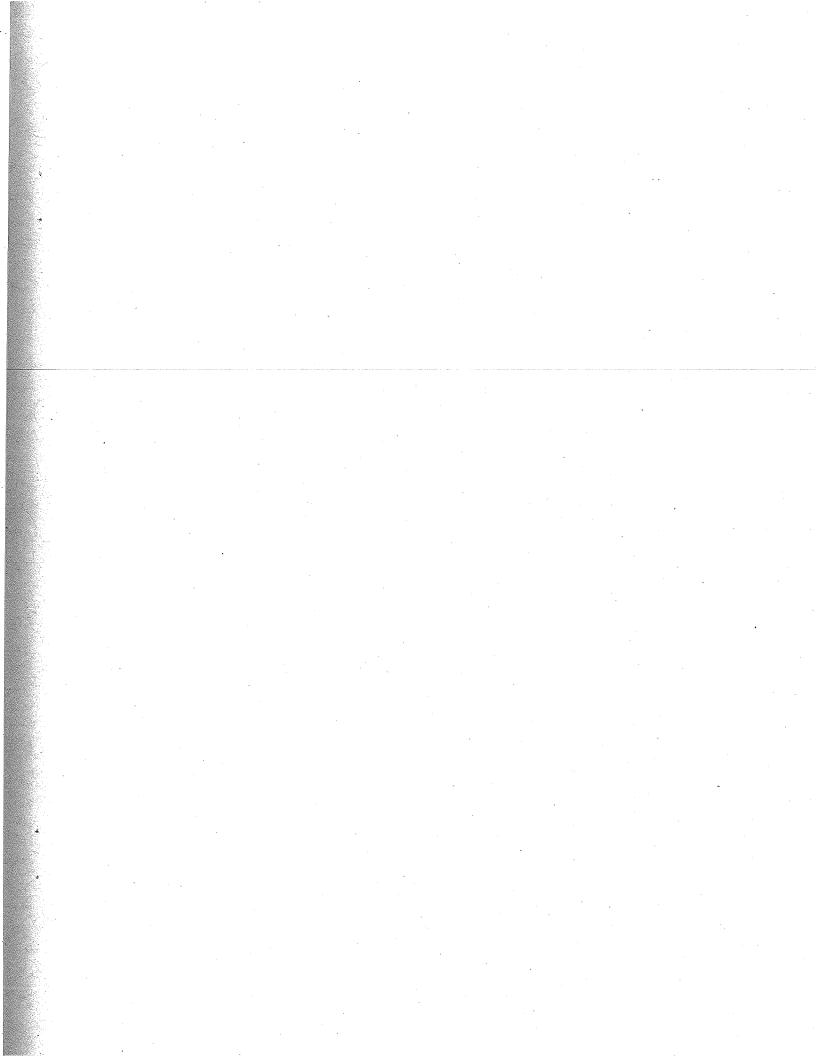
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