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On the Existence and Optimality of Competitive Equilibrium for a Slave Economy¹

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With their fascinating paper, *Economics of Slavery in the Ante-bellum South*, Alfred Conrad and John Meyer [2] have stimulated considerable interest in the question of the institutional stability of slavery. Prior to their work, a conventional view of historians was that slave prices in the antebellum South were inflated well above the capitalized value of slave services. Overcapitalization, it was argued, might have led to the demise of slavery as an institution even in the absence of harsh political action.

Conrad and Meyer undermine the premise of this argument by offering evidence that the prices of slaves in the decade before the Civil War were approximately equal to the present value of their services. The Conrad-Meyer results suggest that there were well functioning competitive markets for slaves during this period.

As the authors acknowledge, their evidence does not provide “a sufficient guarantee of the continuity of southern slavery”. In this paper, a general equilibrium model will be constructed to characterize a competitive economy in which the institution of slavery is present. The existence and welfare properties of an equilibrium for such an economy are investigated. The construction of the model and the theorems derived suggest additional empirical questions which are germane to the issue of the efficiency and institutional stability of slavery.

Data from the 1850 and 1860 censuses are used to lend additional credence to the hypothesis that slavery in the South would not have disappeared due to purely economic forces.

A plausible general equilibrium model of a slave economy requires substantial modification of some assumptions crucial to the traditional theorems on the existence and optimality of competitive equilibrium. In particular, the theorems in this paper extend the results of Arrow, Debreu, and McKenzie [1], [4], [5], [8], [9], to a class of economies where initial wealth allocations are somewhat less rich and diversified than is required for the traditional theorems.

In the construction of the model, the following definitions will be used:

I the set of nonslaves;

J the set of slaves;

M $I \cup J$, the set of all consumers;

K the set of firms;

a_{ij} the fraction of the present value of the j th slave which is owned by the i th non-slave. It is assumed that $\sum_{i \in I} a_{ij} = 1$. That is, the entire present value of the j th slave, as a slave, is owned by members of I ;

¹ First version received April 1969, final version received March 1970 (Eds).

θ_{ik} the fraction of the profits of the k th firm which accrue to i . We assume

$$\sum_{i \in I} \theta_{ik} = 1.$$

All shares in profits of firms are owned by nonslaves.

T_m the feasible trade set of the m th consumer for $m \in M$, T_m is assumed to be a subset of Euclidean n space. $x_m \in T_m$ is a vector of net flows to consumer m . Flows received or emitted at different times are represented by different components of x_m . The restriction to E^n requires that time is divided into discrete periods and that there is a finite (possibly very distant) time horizon. If, for example, consumer m in period t is a net recipient of food and surrenders labour services to others, then the component of x_m corresponding to food in time t has a positive sign while the component corresponding to his own labour in time t has a negative sign.

R_m the preference ordering of consumer m ;

P_m the strict preference relation corresponding to R_m ;

Y_k the production possibility set of the k th firm.

In addition, for $j \in J$ we define $T_j^* \subset T_j$ as the trade set which is feasible for j when j is owned by others. T_j^* need not be identical to T_j if the incentives which can be enforced on j when he is enslaved are insufficient to elicit from him some trades which he could make if he were free.

At any price $p \in E^n$, if the trade vector assigned the j th slave is such as to elicit the highest present value which can be extracted from him so long as he is a slave, then the present value of the j th slave is $\max_{x \in T_j^*} (-px)$.¹ (This is true since by our convention on the signs of trade vectors, negative elements of the vector x_j represent flows *from* j and positive elements represent flows *to* j .)

The main formal difference between a slavery equilibrium, as defined in this paper, and a competitive equilibrium as usually defined is that the pattern of ownership of resources assigns budgets of a somewhat different kind. In particular, since it will be assumed that all consumers are locally nonsatiated, it is in the interest of a slaveowner (as well as his legal right) to extract the maximum possible present value from his slaves. A slave is obliged to surrender to his master a vector of commodities with this maximum value. Since this is the case, the formal treatment of a slave in this model is somewhat similar to that of a firm which is owned by nonslaves. There are, however, important differences.

One difference is that while the production possibility sets of firms are assumed to be convex and to contain the origin, this assumption, as will be shown, seems quite unreasonable for the sets, T_j^* , for $j \in J$.

Furthermore, since the set, T_j^* of trades which can be elicited from slave j is not necessarily equal to the set, T_j , of trades which he can make as a free man, the model will be constructed so as to allow the possibility that a slave may purchase his freedom for a price equal to his present value. For this reason, the preferences of a slave as well as his productivity play a role in the model.

A slave economy will be defined so that at prices, p , any slave, j , is allowed to choose any trade x_j in T_j for which the net cost, px_j , of the trade plus his value to his owner does

¹ We can easily interpret the vector of trades made by a slave to include those of his offspring. In particular, the child of a female slave in the American South was owned by the owner of the mother. (See Meyer and Conrad [2] and Hollander [7]). We could then interpret the trade set of a female slave to include the commodities made available and the commodities consumed by her offspring (with appropriate time designations). The present value of a female slave would thus include the present value of her reproductive capabilities.

not exceed zero. If, for example, \bar{x}_j maximizes $-px$ on T_j^* then $-p\bar{x}_j$ is the value of slave j to his owner. If there is another trade \bar{x}_j in T_j such that $p\bar{x}_j - p\bar{x}_j \leq 0$ and $\bar{x}_j P_j \bar{x}_j$ then the slave may purchase his freedom at the price, $-p\bar{x}_j$, and make the preferred trade, \bar{x}_j . If, on the other hand, there is no way in which j as a free man could offer a trade which cost him no more than $-p\bar{x}_j$ and which he preferred to \bar{x}_j , then there is no way in which he could purchase his freedom *and* make himself better off than he would be as a slave.

The *competitive budgets*, $b(p)$, are defined as follows:

$$\text{If } j \in J, b_j(p) \equiv - \max_{x \in T_j^*} (-px) \equiv \min_{x \in T_j^*} (px).$$

$$\text{If } i \in I, b_i(p) \equiv \sum_{k \in K} \theta_{ik} \max_{y \in Y_k} (py) + \sum_{j \in J} a_{ij} b_j(p).$$

A *slavery equilibrium* is a point, $(\bar{p}, \bar{x}, \bar{y}) \in (E^n, \prod_M T_m, \prod_K Y_k)$ such that:

- (1) For all $k \in K$, $\bar{p}\bar{y}_k \geq \bar{p}y$ for all $y \in Y_k$,
- (2) For all $m \in M$, \bar{x}_m maximizes R_m on $T_m \cap \{x \mid \bar{p}x \leq b_m(\bar{p})\}$,
- (3) $\sum_M \bar{x}_m - \sum_K \bar{y}_k = 0$.

It is also of interest to consider economies in which slaves are not allowed to purchase their freedom and are simply assigned present value maximizing tasks by their masters.¹

A *slave exploitation equilibrium* is a point $(\bar{p}, \bar{x}, \bar{y}) \in (E^n, \prod_I T_i, \prod_J T_j^*, \prod_K Y_k)$ such that conditions (1) and (3) above apply as well as

$$(2') (a) \text{ For } i \in I, \bar{x}_i \text{ is } R_i \text{ maximal on } T_i \cap \{x \mid \bar{p}x \leq b_i(\bar{p})\}.$$

$$(b) \text{ For } j \in J, \bar{p}\bar{x}_j = \min_{x \in T_j^*} (\bar{p}x).$$

The proof of existence offered here, unlike the proofs of McKenzie [9] and Debreu [5] does not employ the assumption that $0 \in T_m$ or that T_m intersects the asymptotic cone of $\sum_K Y_k$. Such an assumption would seem economically unreasonable since if consumer m owns no land or stocks of food and if the production of food requires land then it would be the case that $T_m \cap \sum_K Y_k = \phi$.

One could argue that the origin should be contained in the trade possibility set of any consumer since he could offer the zero trade vector simply by dying immediately. But if for this reason the origin were included in his trade possibility set, it would no longer seem reasonable to assume that this set is convex. Consider a consumer who cannot produce food without trade and who receives barely enough food to survive, while offering some labour. If his trade possibility set were convex and contained the origin, he could offer some labour while receiving only half as much food. This does not seem plausible.

The strategy pursued in this paper will be to assume that T_m is a convex set for all $m \in M$ but not to assume that T_m contains the origin. An economic interpretation of the sets, T_m , can be constructed as follows. Each consumer has a natural lifetime of n_m years. The set, T_m , consists of those trades which he can offer and which enable him to survive for his entire natural lifetime.²

¹ This appears to have been the case in several states in the American South in the 19th century. See Table 1.

² It would, of course, be more satisfactory if the model allowed a natural interpretation for the possibility that the length of life could take values between 0 and n_m ; with an individual's longevity being determined by the trades which he makes. If this is done, it becomes less reasonable to assume that the sets, T_m , are convex. An adequate treatment of this problem would seem to require the use of continuous rather than discrete time.

It will be useful in the ensuing discussion to define the set \bar{T}_m as the convex hull of the set $\{0\} \cup T_m$. That is, $\bar{T}_m \equiv \{x \mid x = \lambda x' \text{ for some } x' \in T_m \text{ and some } \lambda \text{ such that } 0 \leq \lambda \leq 1.\}$

A set, S , of *generally useful bundles* is defined so that S has the following characteristic. If an allocation $x \in \prod_M T_m$ is feasible in the sense that $\sum_M x_m \in \sum_K Y_k$ then the addition of the resources in a vector $s \in S$ to the resources of the conomy would make feasible an allocation z where for all $i \in I$:

(a) If $x_i \in T_i$, then $z_i P_i x_i$;

(b) if $x_i \in \bar{T}_i$ but $x_i \notin T_i$, then either $z_i \in T_i$ or $z_i = x_i$.

Formally, $S \equiv \{s \mid \text{For all } x \in \prod_M T_m \text{ such that } \sum_M x_m = y \text{ for some } y \in \sum_K Y_k \text{ there exists a } z \in \prod_M \bar{T}_m \text{ and a } y' \in \sum_K Y_k \text{ such that } \sum_M z_m = y' + s \text{ and}$

(a) For $i \in I$ such that $x_i \in T_i$, $z_i P_i x_i$,

(b) for $i \in I$ such that $x_i \notin T_i$, either $z_i = x_i$ or $z_i \in T_i\}$.

It may be helpful in interpreting the definition of S to observe that if $0 \in T_i$ for all $i \in I$ then condition (b) becomes superfluous and condition (a) could be written "For all $i \in I$, $z_i P_i x_i$ ". In this case, S consists of all vectors, s , such that if x is a feasible allocation then the addition of the vector s , to the aggregate resources of the economy would make it possible with the use of feasible production processes to produce an aggregate output which could be divided in such a way as to achieve an allocation preferred by every non-slave to the feasible allocation, x .

If, for example, s is a vector of goods such that for all $i \in I$ and all $x_i \in T_i$, $x_i + s P_i x_i$ (and if $\sum_M T_m \cap \sum_K Y_k \neq \phi$) then $s \in S$. The set S also includes vectors, s , such that the resources in s together with existing resources can be transformed by possible production processes into desirable goods.

It may be helpful to consider a simple Ricardian economy. There are three commodities, land, labour, and food. Land is used only for the production of food. There are positive but non-increasing returns to labour in the production of food. Each consumer has a positive minimal food requirement for survival. If he receives at least this minimal requirement he can offer any amount up to some limit, L_m , of labour. It is not difficult to show that if all the land is owned by persons who could survive without trade, then $s \in S$ if the following is true. The vector s has a negative component, $-F$, for food, a zero component for land and a positive component, $+L$, for labour such that when L is used together with the total supply of land and the total potential supply of labour, the incremental output due to the application of L exceeds F .

A key assumption used to prove the existence of a slavery equilibrium is that for all $m \in M$, $-S \cap T_m \neq \phi$. This means that each consumer can offer a vector in S to the rest of the economy and yet survive.

If in the Ricardian economy discussed above, the landholders could add the labour L_m of consumer m to the labour of all other consumers and produce an increment of food greater than the amount, F_m , that he requires to survive, then the vector $(-F_m, L_m, 0) \in S$ and $(+F_m, -L_m, 0) \in T_m$ so that $-S \cap T_m \neq \phi$.

If the model is interpreted so that commodity bundles are distinguished by location as well as by time, the interpretation of the assumption, $-S \cap T_m \neq \phi$, remains fairly reasonable. Observe that it is *not* required that consumer m be able to offer a vector of commodities which itself is desired by every other consumer. It is only assumed that each consumer, m , can offer a trade vector which with the aid of possible production methods (including transportation) and some redistribution of consumption bundles

among individuals makes possible an allocation which would be preferred by any consumer other than m to any allocation possible when consumer m does not make any trades.¹

Theorem 1 can now be stated.

Theorem 1. (a) *A slavery equilibrium exists if:*

- (1) $\sum_K Y_k$ is closed and convex and $\{0\} \in Y_k$ for all $k \in K$;
- (2) For all $i \in I$, T_i is closed and convex. For all $j \in J$, T_j and T_j^* are closed and convex;
- (3) For all $m \in M$, R_m is a continuous, weakly convex, and locally non-satiated quasi-ordering defined on T_m ;
- (4) For all $m \in M$, $T_m \cap \{\sum_K Y_k - \sum_{\substack{i \in M \\ i \neq m}} T_i\}$ is bounded and $\{\sum_I T_i + \sum_J T_j^*\} \cap \sum_K Y_k$ has a nonempty interior;
- (5) There exists a nonzero vector $v \in E^n$ such that for all $m \in M$ and for all $x_m \in T_m$, $x_m + \lambda v R_m x_m$ for all $\lambda \geq 0$;
- (6) For all $m \in M$, $-S \cap T_m \neq \phi$;
- (7) If $j \in J$, $x_j \in$ Boundary T_j and $x'_j P_j x_j$, then $x'_j - x_j \in S$.

(b) *If assumptions 1-6 hold, and if $T_j = T_j^*$ for all $j \in J$, then a slave exploitation equilibrium exists.*

Assumptions 1-4 are familiar from other general equilibrium models (Debreu [4] and [5]). Assumption 5 is satisfied if there is some good which is always harmless. Since this good could also be totally useless, the assumption seems to involve no substantial loss of generality.² Assumption 6 is discussed above. Assumption 7 requires that if the level of preference of a slave is to be raised, he must absorb a bundle of resources desirable to the community. A similar assumption is introduced by Rader (11) under the name of quasi-transferability. Since the proof of Theorem 1 is lengthy and rather intricate, it is confined to the appendix to this paper.

The remaining theorems deal with the optimality properties of slavery equilibrium and slave exploitation equilibrium.

For any partition $\{I, J\}$ of the set, M , of consumers, we say that x is I superior to x' if x and x' are in the set $\prod_M T_m$ and if $x_i R_i x'_i$ for all $i \in I$, and $x_i P_i x'_i$ for some $i \in I$. We say that x is I optimal if $\sum_I x_i \in \{\sum_K Y_k - \sum_J T_j\}$ and if for any x' which is I superior to x , $\sum_I x'_i \notin \{\sum_K Y_k - \sum_J T_j\}$. A feasible allocation, x , is I optimal if there is no allocation feasible for the economy as a whole which is liked at least as well by all members of I and better by some member of I than the allocation x .

Theorem 2. *If preferences are locally nonsatiated and there are no externalities:*

- (1) *A slavery equilibrium is Pareto optimal;*
- (2) *If $T_j = T_j^*$ for all $j \in J$, then a slave exploitation equilibrium is I optimal where I is the set of nonslaves and J is the set of slaves.*

¹ For example, if an additional bushel of potatoes were made available in Maine, it might not be possible to make everyone in the U.S. economy better off by shipping a fraction of a potato to everyone in the U.S. On the other hand, there might be subtle chains of adjustment in production and distribution which could enable everyone to benefit from the availability of a few additional potatoes in Maine. A nice example of this phenomenon is discussed in H. Working's statistical study of potato prices in Minneapolis-St. Paul [16]. See also, Stigler [15].

² In fact, an artificial commodity could be introduced into the commodity space such that the commodity existed in fixed supply and was useless and harmless.

Proof. (1) Consider any allocation, x , Pareto superior to a slavery equilibrium, $(\bar{p}, \bar{x}, \bar{y})$. From the definition of a slavery equilibrium and the assumption of local non-satiation, it follows that $\bar{p}x_m \geq b_m(\bar{p})$ for all $m \in M$ and that $\bar{p}x_m > b_m(\bar{p})$ for some $m \in M$. Therefore $\bar{p} \sum_M x_m > \sum_M b_m(\bar{p})$. But $\sum_M b_m(\bar{p}) = \bar{p} \sum_K \bar{y}_k$. Profit maximization implies that $\bar{p} \sum_K \bar{y}_k \geq \bar{p} \sum_K y_k$ for all $\sum_K y_k \in \sum_K Y_k$. Therefore $\bar{p} \sum_M x_m > \bar{p} \sum_K y_k$ for all $\sum_K y_k \in \sum_K Y_k$. This cannot be the case if $\sum_M x_m \in \sum_K Y_k$. It follows that a slavery equilibrium is Pareto optimal.

(2) If the allocation x is I superior to the slave exploitation equilibrium, $(\bar{p}, \bar{x}, \bar{y})$, then $\bar{p} \sum_I x_i > \sum_I b_i(\bar{p})$. In a slave exploitation equilibrium,

$$\sum_I b_i(\bar{p}) = \sum_K \max_{y \in Y_k} \bar{p}y + \sum_J \max_{x \in T_j^*} (-\bar{p}x).$$

Since, by assumption $T_j = T_j^*$ for all $j \in J$, it must be that

$$\bar{p} \sum_I x_i > \sum_I b_i(\bar{p}) \geq \bar{p} \left(\sum_K y_k - \sum_J x_j \right) \text{ for all } \sum_K y_k - \sum_J x_j \in \sum_K Y_k - \sum_J T_j.$$

It follows that a slave exploitation equilibrium is I optimal.

The next result is a counterpart to the theorem that a competitive equilibrium is in the core of an exchange economy (Debreu and Scarf [6]). As noted by Nikaido [11] and Rader [12], since the production possibility sets of firms are convex and contain the origin, there is no loss of generality in assuming that each consumer, i , can choose any production activity in the (closed, convex) set $\sum_{k \in K} \theta_{ik} Y_k$. For all $i \in I$, define $\hat{Y}_i \equiv \sum_{k \in K} \theta_{ik} Y_k$.

An allocation $(\bar{x}, \bar{y}) \in \left(\prod_M T_m, \prod_I \hat{Y}_i \right)$ is said to be blocked by the coalition, $C \subset M$, if there is an allocation $x \in \prod_C T_c$ such that (a) for all $c \in C$, $x_c R_c \bar{x}_c$; (b) for some $c \in C$, $x_c P_c \bar{x}_c$ and (c) $\sum_C x_c \in \sum_C \hat{Y}_c$.

This means, roughly, that the allocation (\bar{x}, \bar{y}) is blocked by the coalition, C , if the resources which it is physically possible for members of C to offer in trade can be transformed by means of production facilities owned by members of C in such a way as to produce an allocation of resources which each member of C likes at least as well as (\bar{x}, \bar{y}) and some member likes better.

In the following discussion it will be assumed that each slave has only one owner. So long as the property claims of slaveholders are honoured, a slave will not be allowed to enter economic coalitions without the consent of his owner.

A coalition, C , is said to be admissible under slavery obligations if for every slave, j , in the set $C \cap J$, the owner of j also belongs to C .

Theorem 3. *If the assumptions of Theorem 2 are true and if every slave has only one owner, then a slavery equilibrium in which every slave has a non-negative present value cannot be blocked by any coalition which is admissible under slave obligations.*

Proof. Consider a coalition, C , which is admissible under slave obligations. By methods similar to those employed in the proof of theorem 2 it can be shown that if $(\bar{p}, \bar{x}, \bar{y})$ is a slavery equilibrium and if $x_c R_c \bar{x}_c$ for all $c \in C$ and $x_c P_c \bar{x}_c$ for some $c \in C$, then $\bar{p} \sum_C x_c > \sum_C b_c(\bar{p})$. But $\sum_C b_c(\bar{p}) = \sum_{j \in J \cap C} b_j(\bar{p}) + \bar{p} \sum_C \bar{y}_c - \sum_{i \in I \cap C} \sum_{j \in J} a_{ij} b_j(\bar{p})$. Since C is admissible under slave obligations this reduces to

$$\sum_C b_c(\bar{p}) = \bar{p} \sum_C \bar{y}_c - \sum_{i \in I \cap C} \sum_{\substack{j \in J \\ j \notin C}} a_{ij} b_j(\bar{p}).$$

By assumption, the present value, $-b_j(\bar{p})$, of each slave, j , is nonnegative. Hence

$$\bar{p} \sum_c x_c > \sum_c b_c(\bar{p}) \geq \bar{p} \sum_c \bar{y}_c.$$

Profit maximization implies that for all $\sum_c y_c \in \sum_c \hat{Y}_c$, $\bar{p} \sum_c x_c > \bar{p} \sum_c y_c$. Therefore

$$\sum_c x_c \notin \sum_c \hat{Y}_c.$$

This proves Theorem 3.

Another way of stating the result of Theorem 3 is to say that if the economy is at a slavery equilibrium, a slave can enter no coalition which with only the resources available to members of the coalition could make the slave better off without either making other members of the coalition worse off or making his owner worse off.

In general, of course, it would be possible if slavery obligations were abolished for a slave to enter a coalition which does not contain his owner and which blocks a slavery equilibrium.¹

Theorems 2 and 3 suggest that if the economy is at a slavery equilibrium, any improvement in the welfare of slaves could take place only at the cost of a loss of welfare to their owners.

It is nevertheless possible that slavery might disappear as an observable institution even if all slavery contracts were honoured. Our definition of a slavery equilibrium does not require that in equilibrium every member of J remains a slave. All that is required is that if he is to gain his freedom, he must pay his owner a price equal to his present value as a slave.

If the incentive system which can be enforced on a slave is not sufficient to induce him to make some trades which he could make if he were free, it might be that the slave could improve his lot by self purchase.

However, it appears that instances of manumission were rare in the antebellum South. The 1850 and 1860 censuses indicate that fewer than 1/10 of 1 per cent of the slaves received their freedom in each year.² This small percentage could be explained in at least three different ways: (1) Manumission of slaves was outlawed in several states during this period. (2) There were not adequate capital markets or other contractual institutions to allow slaves to finance self-purchase. (3) A slave if freed would be unable to pursue activities which he preferred to slavery and still pay the cost of self-purchase.

Manumission was forbidden in several southern states. Enforcement of these laws was lenient, however. [See Stamp [14] and Matison [10]]. In fact, as indicated by Table I, the rates of manumission were little different whether state law forbade manumission or not. Thus explanation (1) seems inadequate.

Matison cites several interesting examples of methods whereby slaves financed self-purchase. These include illicitly accumulated savings, winnings in a lottery, loans from non-slaves and incentive plans offered by owners or employers. The natural way for a slave to finance his self-purchase would seem to have been to contract with his owner to apply that part of his output beyond some stipulated amount to his self-purchase until he earned his freedom. Slaves, however, had no property rights and in most states, no contract between a slave and his master was binding on the master.³ (Stamp [14],

¹ There are conceivable slavery equilibria for which this is not true. If, for example, a slave belongs to an owner who holds all the land in the economy and if any consumption which he likes at least as well as what he receives as a slave must include some products of the land, then even if he were freed he could enter no coalition which blocks the slavery equilibrium.

² Nor was the percentage of slaves who escaped bondage by flight ever very substantial. The 1850 and 1860 censuses indicate that less than 1/20 of 1% of the slave population escaped in each year. Source: 8th U.S. Census, Vol. 4, p. 338.

³ For example, the Arkansas supreme court ruled that, "If the master contract . . . that the slave shall be emancipated upon his paying to his master a sum of money, or rendering him some stipulated amount of labor, although the slave may pay the money, . . . or perform the labor, yet he cannot compel his master to execute the contract, because both the money and the labor of the slave belong to the master and could constitute no legal consideration for the contract." (Stamp [14], p. 197.)

p. 197; Matison [10]). Three states, Delaware, Louisiana, and Tennessee, enforced written contracts for manumission between master and slave. With the exception of Delaware, which had only a very small number of slaves, these states were little different from the others in their rates of manumission (see Table I).

It thus appears plausible that the majority of slaves could, in fact, be forced under slavery to achieve at least as much productivity as they would if free. This being the case, they simply could not afford to purchase themselves.

It is assumed in theorem 1 that for all consumers, $m \in M$, $-S \cap T_m \neq \phi$. One implication of this assumption is that in equilibrium slaves have a positive present value.

TABLE I
Rates of manumission and legality of manumission in Southern states

States	1850 Census			1860 Census	
	Number of slaves	Per cent manumitted	Manumission prohibited in year	Number of slaves	Per cent manumitted
Alabama	343,000	0.0046	...	435,000	0.0231
Arkansas	47,000	0.0021	1859	111,000	0.0369
Delaware	2,200	12.0960	...	1,800	0.6674
Florida	39,000	0.0559	...	61,745	0.0275
Georgia	382,000	0.0049	1801	463,000	0.0229
Kentucky	211,000	0.0720	...	225,000	0.0780
Louisiana	245,000	0.0649	1851	332,000	0.1558
Maryland	90,000	0.5455	1860	87,000	1.1664
Mississippi	310,000	0.0019	1805	437,000	0.0416
Missouri	87,000	0.0571	...	115,000	0.0774
North Carolina	289,000	0.0006	...	331,000	0.0779
South Carolina	385,000	0.0005	1820	402,000	0.0029
Tennessee	239,000	0.0187	...	276,000	0.0630
Texas	58,000	0.0085	...	183,000	0.0169
Virginia	473,000	0.0461	...	492,000	0.0564
Dist. of Columbia	3,200	0.2514
Total	3,200,000	0.0458	...	3,953,000	0.0763

Sources: Rates of manumission:
8th U.S. Census, Vol. 4, p. 337.
Year of prohibition of manumission, Matison (*op. cit.*).

Available data indicates that adult slaves sold for prices well above zero. Meyer and Conrad have verified that these prices are reasonably accurate reflections of present value. Our assumption, however, would have to be rejected even if infants had non-positive present values.

The institution of slavery might have gradually vanished if infant slaves had been of negative present value. Had this been the case, slave owners would probably have discouraged reproduction or perhaps have engaged in infanticide.

The price of infants can rarely be directly observed since very young children were seldom sold apart from their mothers. The present value of an infant slave can be found indirectly by use of the following formula:

$$V = \sum_{K=1}^{18} P(K) \sum_{T=1}^K \frac{R_T}{(1+r)^T} + \left(1 - \sum_{K=1}^{18} P(K)\right) \left[\sum_{T=1}^{18} \frac{R_T}{(1+r)^T} + \frac{S_{18}}{(1+r)^{18}} \right],$$

where R_T is the net return from a slave of age T , $P(K)$ is the probability that he dies in his K th year, S_{18} is the expected sale price of an eighteen-year-old slave, and r is the interest

rate. V is the sum which an entrepreneur who attempts to maximize the expected present value of his income stream would be willing to pay for an infant slave.

Meyer and Conrad [2] have estimated the prices in 1850 of 18-year-old male and female slaves as \$925 and \$825, respectively. They have also estimated the net returns from slave children of each age. These estimates will be used here for S_{18} and R_T . An important cost in the rearing of young slaves is that a substantial proportion die before they reach their productive years. Death rates for slaves less than ten years of age are not available from the census. We use as a proxy the death rates of Negro children in the District

TABLE II
Present value of infant slaves of each sex at alternative interest rates

Sex	Interest rate		
	5 per cent	8 per cent	12 per cent
Male	\$220	\$100	\$10
Female	\$115	\$25	\$-35

of Columbia in 1901. These are probably not sufficiently different from the rates we seek to substantially alter our conclusions.¹ Table II illustrates the estimated present values of infant slaves calculated for various interest rates.

If one accepts the argument of Meyer and Conrad that the interest rate available to planters was from 6 to 8 per cent in the period considered, it appears that infant slaves of both sexes did have positive present values. At rates of 12 per cent, females take on a negative present value. At slightly higher interest rates this would be the case for both sexes.

TABLE III
Increase in slave population per decade

1810-20	1820-30	1830-40	1840-50	1850-60
33·1	29·1	30·6	23·8	23·4

Source: Bureau of the Census: *Negro Population in the U.S. 1790-1915*.

It thus appears that reproduction of slaves was economically profitable. Corroborating evidence is given by the fact that the slave population grew steadily and substantially during the years after 1808 when the slave trade was eliminated until the Civil War. This is shown by Table III.

In addition to being an efficient instrument of exploitation, the institution of slavery is also very flexible. In fact, *any* subgroup, I , of the economy could employ the institution of slavery to sustain nearly any I optimal pattern of exploitation of the other members of the economy. This result is stated formally in theorem 4.

¹ The percentage of the slave population which died in its second decade of life was reported in the 1860 census to be about 11 per cent. The corresponding percentage for the District of Columbia in 1901 was 10 per cent. Sources: 8th U.S. Census (1860), Vol. 4, p. 337. Bureau of the Census: *United States Life Tables: 1900-1931*, pp. 120-123.

Theorem 4. *If*(1) *For all* $m \in M$,(a) R_m *is convex, continuous and locally nonsatiated,*(b) T_m *is convex and* $T_m \cap -S \neq \phi$,(c) $T_m = T_m^*$;(2) $\sum_K Y_k$ *is convex;*(3) $\sum_M T_m \cap \sum_K Y_k$ *is bounded, closed and has a nonempty interior;*

then for any partition (I, J) *of the population, any* I *optimal allocation* (\bar{x}, \bar{y}) *such that* $\bar{x}_i - s_i \in T_i$ *for some* $s_i \in S$ *for all* $i \in I$ *can be sustained as a slavery equilibrium with some price vector* \bar{p} *and some assignment of claims on profits and the trades made by group* J .

The proof of this theorem, which is a consequence of Minkowski's theorem on the separation of convex sets, will be found in the appendix.

SUMMARY

Theorem 1 displays plausible conditions under which equilibrium exists for a slave economy. The results of Conrad and Meyer indicate that slave prices were not widely divergent from competitive prices. Theorems 2 and 3 suggest that if the economy were at a slavery equilibrium, slavery obligations would be unlikely to disappear simply because of the inefficiency of the system.

In the antebellum South, rates of manumission were low both in states which imposed legal barriers to self-purchase by slaves and in states which did not. The present values of infant slaves were probably non-negative. These results suggest that so long as the ownership of slaves was upheld by the law, the number of persons enslaved would not have substantially diminished either because of self-purchase or because of economically motivated birth control.

The slavery system would appear to be an efficient means of achieving any particular distribution of wealth favourable to the slave owning class. The case against slavery, it seems, must be made not primarily on grounds of inefficiency but on grounds of the morality of the resultant distribution of wealth.

APPENDIX

(A) PROOF OF THEOREM 1

It turns out to be necessary to consider only prices on a half sphere $P \subset E^n$ where

$$P \equiv \{p \mid pv \geq 0\} \cap \left\{ p \mid \sum_{i=1}^n p_i^2 = 1 \right\}$$

where v is the vector whose existence is postulated in assumption 5.

As in Debreu (5), attention will first be confined to choice restricted to bounded subsets of E^n . In particular, choose B^1 a closed bounded cube in E^n such that for all $a \in M$, $T_a \cap \left\{ \sum_K Y_k - \sum_{\substack{m \in M \\ m \neq a}} \bar{T}_m \right\} \subset \text{Int. } B^1$. Assumption 4 ensures that this can be done.

Let $\{B^q\}$ be a sequence of closed bounded cubes such that if $q' \geq q$ then $B^q \subset B^{q'}$ and such that $\lim_{q \rightarrow \infty} B^q = E^n$.

For $m \in M$, let $T_m^q \equiv T_m \cap B^q$ and $\bar{T}_m^q \equiv \text{convex hull } \{0\} \cup T_m^q$. For $j \in J$, let $T_j^{*q} \equiv T_j^* \cap B^q$ and $\bar{T}_j^{*q} \equiv \text{convex hull } \{0\} \cup T_j^{*q}$. For $k \in K$, let $Y_k^q \equiv Y_k \cap B^q$.

Define the budget constraints so that:

$$\text{For } j \in J, \bar{b}_j^q(p) \equiv \min_{x \in T_j^{*q}} (px);$$

$$\text{for } i \in I, \bar{b}_i^q(p) \equiv \sum_{k \in K} \theta_{ik} \max_{y \in Y_k^q} (py) - \sum_{j \in J} a_{ij} \bar{b}_j^q(p).$$

For all $q \geq 1$ and all $k \in K$ define the correspondence $F_k^q: P \rightarrow Y_k^q$ by $F_k^q(p) \equiv \{y \mid y \in Y_k^q \text{ and } py \geq py' \text{ for all } y' \in Y_k^q\}$.

For all $q \geq 1$ and $m \in M$, define $E_m^q: P \rightarrow \bar{T}_m^q$ as follows:

If $p\hat{x} < \bar{b}_m^q(p)$ for some $\hat{x} \in T_m^q$, let $E_m^q(p) = \{x \mid x \text{ maximizes } R_m \text{ on } T_m^q \cap \{x \mid px \leq \bar{b}_m^q(p)\}\}$.

If $px \geq \bar{b}_m^q(p)$ for all $x \in T_m^q$, let $E_m^q(p) \equiv \{x \mid x \in \bar{T}_m^q \text{ and } px = \bar{b}_m^q(p)\}$.

Define $G^q: P \rightarrow E^q \cap B^q$ so that $G^q(p) \equiv \sum_M E_m^q(p) - \sum_K F_k^q(p)$.

Lemma 1. For all $p \in P$ and all $q \geq 1$, (a) $G^q(p)$ is a nonempty convex set and G^q is upper semi-continuous; (b) if $x \in G^q(p)$, then $px \leq 0$.

Debreu [4], shows that for all $k \in K$ and all $p \in P$, $F_k^q(p)$ is nonempty and convex and that F_k^q is u.s.c.

It is easily verified that $\bar{b}_m^q(p) = 0$ for all $m \in M$ and all $p \in P$. Since \bar{T}_m^q is compact and contains the origin, it follows that $\bar{T}_m^q \cap \{x \mid px \leq \bar{b}_m^q(p)\}$ is nonempty and compact for any $q \geq 1$. Since R_m is continuous, $E_m^q(p)$ is nonempty for all $m \in M$ and all $p \in P$. Convexity of $E_m^q(p)$ follows from the weak convexity of preferences.

For all $m \in M$, the function $\bar{b}_m^q(p)$ is the sum of maxima of continuous functions on compact sets and is therefore continuous. If $p\hat{x} < \bar{b}_m^q(p)$ for some $\hat{x} \in T_m^q$ the proof that E^q is u.s.c. at p is the same as that given by Debreu [6]. It is also quite easily verified that if $px \geq \bar{b}_m^q(p)$ for all $x \in T_m^q$, E_m^q is u.s.c. at p . Thus E_m^q is u.s.c. at p for all $p \in P$.

Since, by definition, $G^q(p) \equiv \sum_M E_m^q(p) - \sum_K F_k^q(p)$, part (a) of Lemma 1 follows immediately.

If $x_m \in E_m^q(p)$ for all $m \in M$, then $\sum_M px_m \leq \sum_M \bar{b}_m^q(p)$. But $\sum_M \bar{b}_m^q(p) = \sum_K py_k$ where $y_k \in F_k^q(p)$. It follows that if $\sum_M x_m \in \sum_M E_m^q$ and $\sum_K y_k \in \sum_K F_k^q$, then $p(\sum_M x_m - \sum_K y_k) \leq 0$. This proves part (b) of Lemma 1.

Lemma 2. For all $q \geq 1$, there exists a $\bar{p}(q)$ such that $0 \in G^q(\bar{p}(q))$.

A theorem of Debreu [4], states that if P is the intersection of the unit sphere

$$\left\{ p \mid \sum_{i=1}^n p_i^2 = 1 \right\}$$

and a closed convex cone which is not a linear manifold and if the correspondence, G , maps P into a bounded subset of Euclidean n space and satisfies the conditions of Lemma 1, above, then there is a $\bar{p} \in P$ such that $G(\bar{p}) \cap -P^* \neq \emptyset$ where $-P^* \equiv \{-x \mid px \geq 0 \text{ for all } p \in P\}$.

Since $\{p \mid pv \geq 0\}$ is a closed half space, and for all $q \geq 1$, G^q satisfies Lemma 1, there is a $\bar{p}(q)$ such that $G^q(\bar{p}(q)) \cap -P^* \neq \emptyset$. From the duality theorem for closed convex cones (Nikaido [11]) it follows that $-P^* \equiv -\{\lambda v \mid \lambda \geq 0\}$. Therefore for some $\bar{p}(q)$ and some $\bar{\lambda} \geq 0$, $-\bar{\lambda}v \in G^q(\bar{p}(q))$.

It will now be shown that for $\bar{p} = \bar{p}(q)$, $0 \in G^q(\bar{p})$.

Consider a point $\bar{x} \in \prod_M \bar{T}_m^q$ such that $\bar{x}_m \in E_m^q(\bar{p})$ for all $m \in M$ and such that for some $\bar{y} \in \sum_K F_k^q(\bar{p})$, $\sum_M x_m - \bar{y} = -\bar{\lambda}v$ where $\bar{\lambda} \geq 0$. It is not hard to show that for any $a \in M$, $\bar{x}_a + \bar{\lambda}v \in \bar{T}_a$. Since $\bar{x}_a + \bar{\lambda}v + \sum_{\substack{m \in M \\ m \neq a}} \bar{x}_m = \bar{y}$, it follows from the definition of B^q that $\bar{x}_m \in \text{Int } B^q$ for all $m \in M$ and that $\bar{x}_m + \bar{\lambda}v \in \text{Int } B^q$ for all $m \in M$. Since this is the case, the assumption of local nonsatiation can be used to show that $\bar{p}\bar{x}_m = \bar{b}_m^q(\bar{p})$ for all $m \in M$. Therefore $\bar{p} \sum_M \bar{x}_m = \sum_M \bar{b}_m^q(\bar{p}) = \bar{p}\bar{y}$. But $\sum_M \bar{x}_m + \bar{\lambda}v = \bar{y}$. It must be that $\bar{p}\bar{\lambda}v = 0$. For some $a \in M$, let $\hat{x}_a = \bar{x}_a + \bar{\lambda}v$. Then $\bar{p}\hat{x}_a = \bar{p}\bar{x}_a$. From the definition of v it follows that $\hat{x}_a \in E_a^q(\bar{p})$. But $\hat{x}_a + \sum_{\substack{m \in M \\ m \neq a}} \bar{x}_m = \bar{y}$. Therefore $0 \in G^q(\bar{p})$.

Lemma 3. *There exists a point $(\bar{p}, \bar{x}, \bar{y}) \in (P, \prod_M \bar{T}_m, \prod_K Y_k)$ such that:*

(1) $\bar{p}\bar{y}_k \geq \bar{p}y$ for all $y \in Y_k$;

(2) For all $m \in M$,

(a) \bar{x}_m maximizes R_m on $\{x \mid \bar{p}x \leq b_m(\bar{p})\} \cap T_m$ if $\min_{x \in T_m} \bar{p}x < b_m(\bar{p})$,

(b) $\bar{p}\bar{x}_m = b_m(\bar{p})$ if $\min_{x \in T_m} \bar{p}x \geq b_m(\bar{p})$;

(3) $\sum_M \bar{x}_m = \sum_K \bar{y}_k$.

Proof. Lemma 2 guarantees that there is a sequence $\{p(q), x(q), y(q)\}$ such that $(p(q), x(q), y(q)) \in (P, \prod_M \bar{T}_m^q, \prod_K Y_k^q)$ and the following conditions are satisfied for all $q \geq 1$.

(1) For all $k \in K$, $y_k(q) \in F_k^q(p(q))$;

(2) For all $m \in M$, $x_m(q) \in E_m^q(p(q))$;

(3) $\sum_M x_m(q) = \sum_K y_k(q)$.

Condition (3), together with Assumption 4 of Theorem 1 guarantees that for all $q \geq 1$, $(p(q), x(q), y(q))$ belongs to the compact set $\{P, \prod_M \bar{T}_m^1, \prod_K Y_k^1\}$.

Therefore for some $(\bar{p}, \bar{x}, \bar{y}) \in \{P, \prod_M \bar{T}_m, \prod_K Y_k\}$, $(p(q), x(q), y(q)) \rightarrow (\bar{p}, \bar{x}, \bar{y})$. It is a straightforward albeit somewhat tedious exercise to show that $(\bar{p}, \bar{x}, \bar{y})$ satisfies the conditions of Lemma 3. Q.E.D.

Lemma 4. *If $s \in S$, and $(\bar{p}, \bar{x}, \bar{y})$ satisfies Lemma 3, then $\bar{p}s > 0$.*

Proof. The definition of S states that if $s \in S$, there exists a $z \in \prod_M \bar{T}_m$ such that $\sum_M z_m = \sum_K \bar{y}_k + s$ and if $\bar{x}_m \in T_m$ then $z_m P_m \bar{x}_m$, if $\bar{x}_m \notin T_m$ then $z_m = \bar{x}_m$. It is readily verified that in either case $\bar{p}z_m \geq \bar{p}\bar{x}_m$.

Assumption 4 of Theorem 1 implies that $\sum_M T_m \cap \sum_K Y_k$ has a nonempty interior. It follows that for all $p \in P$, there is some $a \in M$ such that $\min_{x \in T_a} \bar{p}x < b_a(\bar{p})$. Since this is the case and since $z_a P_a \bar{x}_a$, it must be that $\bar{p}z_a > \bar{p}\bar{x}_a$. (See Debreu [4]). Therefore,

$$\sum_M \bar{p}z_m > \sum_M \bar{p}\bar{x}_m.$$

But $\sum_M \bar{p}z_m = \bar{p} \sum_M z_m = \bar{p} \sum_M \bar{x}_m + \bar{p}s$. Hence $\bar{p}s > 0$. Q.E.D.

Since $0 \in Y_k$ and $0 \in \bar{T}_j$ for all $k \in K$ and all $j \in J$, $b_i(p) \geq 0$ for all $i \in I$ and all $p \in P$. According to Lemma 4 and Assumption 6, $\bar{p}s > 0$ for all $s \in S$ and $-S \cap T_i \neq \phi$. Therefore $\min \bar{p}x < 0 \leq b_i(p)$ for all $i \in I$. The fact that $\bar{x}_i \in E_i(\bar{p})$ implies that \bar{x}_i maximizes R_i on $T_i \cap \{x \mid \bar{p}x \leq b_i(\bar{p})\}$ for all $i \in I$.

Part (a) of Theorem 1 will follow if it can be shown that at \bar{x} , preferences of slaves are also maximized subject to $\bar{p}x_j \leq b_j(p)$.

If for $j \in J$, $\min_{x \in T_j} \bar{p}x < b_j(\bar{p})$, it is immediate that \bar{x}_j maximizes

$$R_j \text{ on } T_j \cap \{x \mid \bar{p}x \leq b_j(\bar{p})\}.$$

If $\min_{x \in T_j} \bar{p}x \geq b_j(\bar{p})$, it will be shown that $\bar{x}_j \in \text{Boundary } T_j$.

Since $T_j^* \cap -S \neq \phi$, $b_j(\bar{p}) \equiv \min_{x \in T_j^*} (\bar{p}x) < 0$. Since $\bar{x}_j \in E_j(\bar{p})$, $\bar{p}\bar{x}_j = b_j(\bar{p})$, and $\bar{x}_j \neq 0$.

Suppose $\bar{x}_j \in T_j$. Since $\bar{x}_j \in \bar{T}_j$, $\lambda \bar{x}_j \in T_j$ for some $\lambda > 1$. But $\bar{p}\bar{x}_j = b_j(\bar{p}) < 0$. Therefore $\bar{p}(\lambda \bar{x}_j) < b_j(\bar{p})$. Hence $\min_{x \in T_j} \bar{p}x < b_j(\bar{p})$. It follows that if $\min_{x \in T_j} \bar{p}x \geq b_j(\bar{p})$, then $\bar{x}_j \in T_j$ and that $\bar{x}_j \in \text{Boundary } T_j$.

According to Assumption 7, if $x_j P_j \bar{x}_j$, then $x_j = \bar{x}_j + s$ for some $s \in S$. But $\bar{p}s > 0$. Therefore, if $x_j P_j \bar{x}_j$ then $\bar{p}x_j > \bar{p}\bar{x}_j$.

It follows that for all $j \in J$, \bar{x}_j maximizes R_j on $T_j \cap \{x \mid \bar{p}x \leq b_j(\bar{p})\}$. This proves part (a) of Theorem 1.

It was shown, using only assumptions 1-6, that for all $i \in I$, \bar{x}_i maximizes R_i on $T_i \cap \{x \mid \bar{p}x \leq b_i(\bar{p})\}$, and that for all $j \in J$, $\bar{p}\bar{x}_j = \min_{x \in T_j^*} \bar{p}x$ and $\bar{x}_j \in T_j$. If for all $j \in J$, $T_j = T_j^*$, then $\bar{p}\bar{x}_j = \min_{x \in T_j} \bar{p}x$ and $\bar{x}_j \in T_j^*$. This proves part (b) of Theorem 1.

Q.E.D.

B. PROOF OF THEOREM 4

Let $\hat{P}_I(\bar{x}) = \{\sum_I x_i \mid x_i \in T_i \text{ and } x_i R_i \bar{x}_i \text{ for all } i \in I, x_i P_i \bar{x}_i \text{ for some } i \in I\}$. If (\bar{x}, \bar{y}) is I optimal, then $\hat{P}_I(\bar{x}) \cap \{\sum_K T_i - \sum_J Y_j\} = \phi$. Since preferences are convex, $\hat{P}_I(\bar{x})$ is convex. Also $\{\sum_K Y_k - \sum_J T_j\}$ is the sum of the convex sets and is therefore convex. According to Minkowski's theorem, there exists a vector, \bar{p} , such that $\sum_I x_i \in \hat{P}_I(\bar{x})$ and

$$\sum_K y_k - \sum_J x_j \in \{\sum_K Y_k - \sum_J T_j\}$$

implies that $\bar{p} \sum_I x_i \leq \bar{p}(\sum_K y_k - \sum_J x_j)$. Since $\sum_I \bar{x}_i = \sum_K \bar{y}_k - \sum_J \bar{x}_j$, it follows that at prices \bar{p} , \bar{y}_k is a profit maximizing output for $k \in K$ and x_j maximizes $-\bar{p}\bar{x}_j$ on the set T_j^* (by assumption, $T_j = T_j^*$).

Lemma 4, above, can be employed to show that for all $s \in S$, $\bar{p}s > 0$. Since, by assumption, for all $i \in I$, $\bar{x}_i - s_i \in T_i$ for some $s_i \in S$ and since $\bar{p}(\bar{x}_i - s_i) = \bar{p}\bar{x}_i - \bar{p}s_i < \bar{p}\bar{x}_i$, \bar{x}_i is not a cost minimizing trade in T_i . Therefore for all $i \in I$, $x_i P_i \bar{x}_i$ implies $\bar{p}x_i > \bar{p}\bar{x}_i$. (See Debreu [4]).

Collecting results, we have:

- (1) $y \in Y_j$ implies $\bar{p}y \leq \bar{p}\bar{y}_j$;
- (2) For all $i \in I$, \bar{x}_i maximizes R_i on $\{x \mid \bar{p}x \leq \bar{p}\bar{x}_i\}$,

$$\text{for } j \in J, \bar{p}\bar{x}_j = \min_{x \in T_j} \bar{p}x < 0.$$

Since (\bar{x}, \bar{y}) is I optimal,

$$(3) \sum_M \bar{x}_m = \sum_K \bar{y}_k.$$

If all members of I are given budgets, $b_i(\bar{p}) = \bar{p}\bar{x}_i$, then according to (1), (2), and (3), $(\bar{p}, \bar{x}, \bar{y})$ is a slave exploitation equilibrium. Since $\bar{p} \sum_M \bar{x}_i = \bar{p} \sum_K \bar{y}_k - \bar{p} \sum_J \bar{x}_j$, one can assign claims on profits and the present value of slaves in such a way that $b_i(\bar{p}) = \bar{p}\bar{x}_i$ and $\sum_I b_i(\bar{p}) = \bar{p} \sum_I \bar{x}_i = \bar{p} \sum_K \bar{y}_k - \bar{p} \sum_J \bar{x}_j$. This establishes Theorem 4.¹

REFERENCES

- [1] Arrow, K. J. and Debreu, G. "Existence of an Equilibrium for a Competitive Economy", *Econometrica*, **22** (1954), 265-290.
- [2] Conrad, A. H. and Meyer, J. R. "The Economics of Slavery in the Antebellum South", *Journal of Political Economy*, **66** (1958), 95-122.
- [3] Debreu, G. "Market Equilibrium", *Proceedings of the National Academy of Sciences*, **42** (1956), 876-878.
- [4] Debreu, G. *Theory of Value* (New York, John Wiley and Sons, 1959).
- [5] Debreu, G. "New Concepts and Techniques for Equilibrium Analysis", *International Economic Review*, **3** (Sept. 1962).
- [6] Debreu, G. and Scarf, H. "A Limit Theorem on the Core of an Economy", *International Economic Review*, **4** (Sept. 1963).
- [7] Hollander, B. *Slavery in America: Its Legal History* (London, Bowes & Bowes, 1962).
- [8] McKenzie, L. W. "On the Existence of General Equilibrium: for a Competitive Market", *Econometrica*, **27** (1959), 54-57.
- [9] McKenzie, L. W. "On the Existence of General Equilibrium: Some Corrections", *Econometrica*, **29** (1961), 247-248.
- [10] Matison, S. E. "Manumission by Purchase", *Journal of Negro History*, **33** (1948), 146-167.
- [11] Nikaido, H. *Convex Structures and Economic Theory* (New York, Academic Press, 1968).
- [12] Rader, J. T. "Edgeworth Exchange and General Economic Equilibrium", *Yale Economic Essays*, **4** (1964).
- [13] Rader, J. T. "Pairwise Optimality and Non-Competitive Behavior", in *Papers in Quantitative Economics*, ed. J. P. Quirk and A. M. Zarley (Lawrence, Kansas, University of Kansas Press, 1968), 101-128.
- [14] Stamp, K. M. *The Peculiar Institution* (New York, Alfred A. Knopf, 1956).
- [15] Stigler, G. *The Theory of Price*, Third Edition (New York, Macmillan, 1966).
- [16] Working, H. "Factors Determining the Price of Potatoes in St Paul and Minneapolis", *Technical Bulletin* 10, University of Minnesota Agricultural Experiment Station (1922).

¹ It might be useful to change the definition of I optimal to allow the possibility that members of J be allowed to starve. If this is the case, one would replace T_j by $T_j \cup \{0\}$ everywhere in the definition of I optimality. If $\{0\} \notin X_j$ then these sets are non-convex. With the assumption that $-S \cap T_j \neq \phi$, however, it is easy to show that if (\bar{x}, \bar{y}) is I optimal, then for all $j \in J$, $\bar{x}_j \neq 0$. The same hyperplane p used in Theorem 4 will still sustain (\bar{x}, \bar{y}) as a slave exploitation equilibrium.