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RISKY BUSINESS:  
THE ALLOCATION OF CAPITAL

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Abstract

This paper examines the effect of risk on the firm's demand for capital and the equilibrium allocation of capital. Capital is an asset the firm uses to transfer sales between periods and an asset society uses to transfer consumption between periods. The firm diversifies risk through the mix of factor inputs, and an increase in price risk can make capital a more valuable asset to the firm, increasing the firm's investment demand. Society diversifies risk across production technologies. In a simple general equilibrium model I show that an increase in exogenous risk causes a reallocation of capital among technologies, but does not increase aggregate investment.

JEL Classification: 023, 313



## RISKY BUSINESS: THE ALLOCATION OF CAPITAL

## Introduction:

This paper examines the effect of risk on the firm's demand for capital and the effect of risk on the equilibrium allocation of capital. Economists have studied the effect of risk on agents' decisions and the allocation of resources for long time. This is a difficult problem at best, and frequently intractable. General equilibrium theorists prove the existence of an equilibrium under risk, but they do not examine how changes in risk would affect agents' behavior. Hartman (1972), Pindyck (1982), and Abel (1983, 1984, 1985), in a partial equilibrium setting focusing on firm behavior, were able to show that a mean-preserving spread in the price of output increased the firm's investment demand.

At first glance the Hartman-Pindyck-Abel result seems surprising. A mean-preserving spread in the price of output increases the variability of the firm's future revenue stream, which would appear to increase the riskiness of investment. In this paper, I show that the firm diversifies risk through the mix of factor inputs. A mean-preserving spread in the price of output increases the diversification value of capital to the firm, and the firm's demand for capital; in the jargon of finance, capital is a "negative beta" asset. In a general equilibrium, prices and the quantity of capital are endogenous. The random "state of

nature" is exogenous to the economy. Society diversifies risk across production technologies. In a simple general equilibrium model, I show that the partial equilibrium result for the firm does not extend to the equilibrium aggregate quantity of capital. A mean-preserving spread in the distribution of the state of nature causes a reallocation of capital among production technologies, but it does not increase equilibrium aggregate investment.

Modern finance theory values financial assets in terms of their contribution to the household's expected utility. An asset that has a high payoff when the marginal utility of consumption is high contributes more to expected utility than an asset whose payoff is independent of marginal utility. For example, in the traditional capital-asset pricing model, an asset whose return is negatively correlated with the market return (a negative beta asset) adds more to a risk averse agent's expected utility than the asset's expected return since it reduces the portfolio's risk. For the firm, capital is an asset and the marginal product of capital is the "payoff" to the asset. Capital's marginal contribution to expected revenue depends on the payoff to capital times the price--the marginal revenue product of capital. Labor variation makes capital's marginal contribution to expected revenue an increasing function of price risk. When prices are high the firm hires more labor,

increasing the marginal physical product of capital, and vice versa. Capital has a high marginal payoff when the revenue value of the payoff is high. Intuitively, the ability to vary labor with the price of output makes capital a negative beta asset, so that an increase in price risk increases the asset value of capital to the firm.

In a general equilibrium risk depends on the exogenous random state of nature, which affects either households' preferences, firms' technologies, or both. Capital is an asset society uses to transfer consumption between periods. To examine the effect of risk on the equilibrium allocation of capital, I use a simple dynamic general equilibrium model with a closed form solution. The model is based on Brock (1982), and Long & Plosser (1983). In this model, an increase in price risk, ceteris paribus, would increase the demand for capital; but a mean-preserving spread in the distribution of the state of nature has no effect on equilibrium aggregate investment. An increase in exogenous risk does cause the reallocation of capital among production technologies, since society can diversify the nonsystematic risk.

Closed form solutions to general equilibrium models rest on very special functional forms, so it is dangerous to generalize from a specific model. Nevertheless, the model

fits (my) intuition that an increase in nondiversifiable risk should not cause society to sacrifice additional current consumption for riskier future consumption. While some models might give this result, the example in this paper shows it is not a general result.

### Section 1: A Model of Firm Behavior

This section presents a simple model of the firm that highlights the effect of labor variation on capital's contribution to expected revenue. Essentially the firm diversifies risk by combining a fixed input, capital, with a variable input, labor. A mean-preserving spread in the price of output increases capital's contribution to expected revenue, and the firm's investment demand.

The objective of the firm is to maximize the discounted expected value of the revenue stream,

$$1.1 \quad \sum_{j=0}^{\infty} R^j E_t [p_{t+j} q_{t+j}]$$

where  $p$  denotes the price of output,  $q$  the net quantity of sales, and the product,  $pq$ , net revenue.  $R$  is the nominal discount factor, ie, the reciprocal of one plus the nominal rate of interest.<sup>1</sup> The prices and discount factor are exogenous to the firm. At a maximum the firm cannot increase the value of the objective function by transferring revenue between periods,

$$1.2 \quad d(p_t q_t) = R E_t [d(p_{t+1} q_{t+1})]$$

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<sup>1</sup> The interest rate can be time-varying, but not random.



the discounted increase in expected future revenue from the marginal transfer just equals the decrease in current revenue.

The firm rearranges the temporal pattern of revenue through sales. Define the quantity of net sales,  $q$ , as,

$$1.3 \quad q_t = f(k_{t-1}, z_t, s_t) - (P_t/p_t)\{k_t - k_{t-1}\} - (w_t/p_t)z_t$$

output,  $f(\dots)$ , minus the real cost of investment,

$$(P/p)_t\{k_t - k_{t-1}\}, \text{ minus real payments to labor, } (w/p)z.$$

Output is a concave function of the fixed (predetermined) factor, capital ( $k$ ), a variable factor, labor ( $z$ ), and a random shock ( $s$ ).

I make much stronger assumptions than one needs to show that a mean-preserving spread in the price of output increases investment.<2> My purpose is to make the role of labor variation on capital's value as an asset transparent. I assume the production function is Cobb-Douglas,

$$1.4 \quad f(k_{t-1}, z_t, s_t) = k_{t-1}^a z_t^b s_t \quad ; \quad a+b < 1.$$

For the stochastic environment, I assume the price of output,  $p_t$ , the price of capital,  $P_t$ , the wage,  $w_t$ , and the state of nature,  $s_t$ , are independently and identically distributed random variables.

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2 For example see Hartman, Pindyck, or Abel. They use models of the firm with a linearly-homogeneous production function and a cost to adjusting capital. The appendix extends the the analysis in this section to a model with a cost to adjusting capital.

The firm chooses its labor input each period so that,

$$1.5 \quad f_{z,t} = b k_{t-1}^a z_t^{b-1} s_t = (w/p)_t$$

the marginal product of labor equals the real wage. Labor demand is a decreasing function of the wage and an increasing function of current price of output and productivity shock,

$$1.6 \quad z_t = f_z^{-1}(w/p_t, k_{t-1}, s_t) \\ = (w_t/p_t s_t)^{-1/(1-b)} k_{t-1}^{a/(1-b)} \text{constant.}$$

Labor is a variable input that the firm selects after it observes the realization of the random variables.

Capital is an asset that allows the firm to transfer sales between periods. The firm must choose capital before it observes the realization of the random variables. The value of capital depends on the risky future payoffs. The firm invests each period until,

$$1.7 \quad P_t = RE_t [p_{t+1} f_{k,t} + P_{t+1}].$$

the discounted value of the expected marginal revenue product of capital plus the expected market value of a unit of capital next period equals the current price of capital. Equation 1.7 is an asset valuation equation. It measures the effect of transferring revenue between periods with an asset purchase. The left-hand side gives the loss in current revenue from purchasing an additional unit of capital. The right-hand side gives the discounted expected gain in

revenue from owning an additional unit of capital. The expected gain consists of the payoff, or dividend, from the asset, plus the value of the asset.

Equation 1.7 is analogous to the household asset evaluation equation in the consumption-capital-asset pricing model (CCAPM). In the CCAPM the household transfers consumption between periods through financial assets to maximize the present value of expected lifetime utility. The transition equation for a financial asset is,

$$1.8 U_{c,t} V_t = D E_t [U_{c,t+1} \{V_{t+1} + d_{t+1}\}],$$

where  $U_c$  denotes the marginal utility of consumption and  $D$  the household time-discount factor.  $V$  is the price of the asset and the  $d$  the dividend. The left-hand side measures the decrease in current utility from purchasing an additional unit of the asset--the price of the asset,  $V$ , times the marginal utility of the lost consumption. The right-hand side measures present value of the increase in expected utility from owning the asset--the payoff, or dividend, plus the value of the asset measured in consumption units, times the marginal utility of the additional units of consumption.

Viewing equation 1.7 as an asset demand equation helps one interpret the effects of risk on the firm's demand for capital. Households diversify risk by holding a portfolio of

assets whose payoffs are not perfectly correlated. An asset whose return is negatively correlated with the portfolio return is (usually) more valuable because it has high payoffs when the portfolio payoff is low. An asset that has high payoffs when household consumption is low, and the marginal utility of consumption high, contributes more to household utility than the expected value of the payoff. "Negative beta" assets diversify risk. The firm diversifies risk with the mix of factor inputs.

Risk affects investment demand through the expected marginal revenue product of capital. Solving the first-order difference equation 1.7 forward by recursively eliminating  $P_{t+1}, P_{t+2}, \dots$ , expresses,

$$1.9 \quad P_t = \sum_{j=1}^{\infty} R^{-j} E_t p_{t+j} f_{k,t+j-1}$$

the firm's demand for capital as the present value of the expected stream of marginal revenue that the investment generates. At a maximum, the present value of the expected asset payoffs equals the cost of the asset.

If the marginal revenue product of capital is convex in a random variable, then a mean-preserving spread in the random variable increases the expected marginal revenue product of capital, and capital's contribution to expected revenue.

Substituting equation 1.6 for labor in the marginal revenue product of capital and rearranging gives,

$$1.10 \quad p_t f_k(k_{t-1}, z_t, s_t) \\ = p_t [p_t^{b/(1-b)} w_t^{-b/(1-b)} k_{t-1}^{(a+b-1)/(1-b)} s_t^{1/(1-b)}]$$

where I left off a constant.

Equation 1.10 is convex in the price of sales,  $p$ , so a mean-preserving spread in the price increases the expected marginal revenue product of capital. At the time the firm makes its investment decision it does not know the future price of output. It does know, however, that if the price of output is high, *ceteris paribus*, the labor input will be high, so the marginal physical product of capital will be high. The physical payoff to investment is high in states of nature where the revenue value of the additional units of sales (the price) is high. The firm "diversifies" risk by combining capital with a variable input that moves in the same direction as the price of output. A mean-preserving spread in the price of output increases labor variation, and increases the diversification value of capital.

A mean-preserving spread in wages also increases capital's contribution to expected revenue, since it increases the expected labor input (see equation 1.6) and the expected marginal physical product of capital. If real wages are fixed (wages and prices are perfectly correlated), however,

a mean-preserving spread in the random variable has no effect on investment demand, since the firm cannot diversify risk through labor variation. <3> When the variables are correlated, the effect of risk on the firm's investment demand depends on the correlation, eg see Abel (1985).

General results about the effect of changes in risk on firm, or household, behavior are hard to find. Most of our intuition comes from linear models like the capital-asset pricing model. For example, suppose we decompose the expectation of the product of the random variables, <4>

$$1.14 \quad E[U_c d] = EU_c E d + \text{cov}(U_c, d),$$

into the product of the expectations plus their covariance.

If the marginal utility of consumption and the asset payoffs are linear functions of the random variable, then the covariance term neatly summarizes the asset's risk. Let,

$$1.15 \quad \beta(d) = -\text{cov}(U_c, d) = \rho(U_c, d) e(U_c) e(d),$$

denote the asset's risk; the so-called consumption beta. If the asset's return is negatively correlated with consumption [ $\rho(cd) < 0$ ], so it is positively correlated with the marginal utility of consumption, then the asset contributes more to household expected utility than the expected value

3 This example provides a rationalization for firms' preferences for fixed nominal wage contracts. If wages and prices are correlated it reduces the firm's ability to diversify risk through labor variation.

4 Notice,  $U_{c,t} V_t = DE_t [V_{t+1} + dt_{+1}] = dD^j E_t [U_{c,t+j} dt_{+j}]$ , so equation 1.14 is a decomposition of an element of the sum.

of its payoff. Now a mean-preserving spread in the random variable (notice that this is equivalent to a mean-preserving spread in the price of output in the model of the firm)--increases the value of the asset to the household. A "negative beta" asset helps diversify risk. An increase in risk makes this asset more valuable.

When the random variable enters nonlinearly, the covariance is not a sufficient statistic for risk evaluation. The effect of risk on the value of the asset depends on the convexity, or concavity, of the function. A convex function is analogous to a negative beta; ceteris paribus, an increase in risk makes negative beta assets more valuable.

## 2: A Simple General Equilibrium Model

In financial asset pricing models, households treat the prices of assets as exogenous, and their behavior determines the quantity demanded. In asset market equilibrium, the quantity of financial assets and the distribution of asset payoffs are exogenous; the household demands determine the asset prices. For a competitive firm, output and factor prices are exogenous, but prices and wages are endogenous in the market equilibrium. In general equilibrium, output and the decomposition of output into consumption and investment are endogenous. The firms' technologies and investment decisions constrain household consumption, and the households' labor supply decisions constrain the firms. Financial assets are simply claims on real capital and its productive potential. Random shocks--the so-called state of nature--are exogenous to the economy. The endogenous prices, which individual agents view as exogenous, are complicated functions of the state of nature.

This section examines the effect of risk on the equilibrium allocation of capital in a simple general equilibrium model with a closed form solution. The model and solution are based on the work of Brock (1982), and Long & Plosser (1983). In this model a mean-preserving spread in the



distribution of the state of nature has no effect on current aggregate investment. I present two examples. The first has a single aggregate production technology which provides no opportunity to diversify risk. The second has multiple production processes, and a change in risk affects the allocation of capital across technologies, but not aggregate investment. These examples fit my intuition that an increase in nondiversifiable risk should not increase investment.

#### The Model: A Single Production Process

Consider an economy with a representative household and firm. The household maximizes expected (infinite) lifetime utility,

$$2.1 \sum D^j E_t U(c_{t+j}, l-z_{t+j}, s_{t+j})$$

where  $D$  is the household time discount factor,  $c$  is consumption,  $l-z$  is leisure, and  $z$  labor, and  $s$  is an independently and identically distributed random shock. I refer to  $s$  as the state of nature. The household's instantaneous concave utility function is,

$$2.2 U(\dots) = \ln(c_t) + u(l-z_t, s_t)$$

The firm's technology and investment decision constrain household consumption,

$$2.3 c_t = f(k_{t-1}, z_t, s_t) - \{k_t - k_{t-1}\}.$$

I assume the production function is homogeneous of degree one in capital ( $k$ ) and labor, and that labor and capital are complementary factors of production, ie  $f_{kz} > 0$ .

In an equilibrium, the marginal product of labor equals the "real wage",

$$2.4 \quad f_{z,t} = -U_{z,t}/U_{c,t},$$

ie, the marginal disutility of labor relative to the marginal utility of consumption. The equilibrium quantity of labor is determined each period after the realization of the shock. Let,

$$2.5 \quad z^*_t = f_z^{-1}(k_{t-1}, s_t, U_{z,t}/U_{c,t})$$

denote the equilibrium quantity of labor.

Capital is the asset society uses to transfer consumption between periods. Society must choose its capital allocation before it observes the realizations of the state of nature.

Capital is a risky asset. In equilibrium, society invests until,

$$2.6 \quad U_{c,t} = DE_t[U_{c,t+1}\{f_{k,t+1}\}]$$

the decrease in current utility from the last unit of investment equals the discounted increase in expected future utility of having an additional unit of capital.

In Section 1 I examined a case where the marginal revenue product of capital was convex in the price of output, so a mean-preserving spread in the price of output increases the firm's demand for capital. The marginal "revenue product" of capital in this section also is convex in the marginal

utility of consumption (the shadow price of sales).

Following Abel (1984), we can use the fact that in linearly homogeneous production functions, the marginal physical product of capital evaluated at the optimal labor input ( $z^*$ ),

$$2.7 \quad f_k(k_{t-1}, z^*_t, s_t) = g'(s_t, U_{z,t}/U_{c,t}) = g(s_t)$$

is independent of the capital stock. Abel shows that the marginal revenue product of capital,  $U_c g'()$ , is convex in the marginal utility of consumption  $\langle 5 \rangle$ . Thus, a mean-preserving spread in the marginal utility of consumption, ceteris paribus, increases capital's contribution to expected utility. To make the case for investment demand increasing with risk even stronger, assume that  $g(s)$ , the marginal physical product of capital, is convex in  $s$ .  $\langle 6 \rangle$  Therefore, a mean-preserving spread in the state of nature increases the expected marginal physical product of capital.

In general equilibrium, the marginal utility of consumption (the price of consumption) is endogenous. A mean-preserving spread in the exogenous state of nature, in general, will not map into a mean-preserving spread in the marginal utility of consumption, and it will not increase capital's

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5 Abel shows that the marginal revenue product of capital is convex in the price of sales, and the price of sales is proportional to the marginal utility of consumption in equilibrium.

6 In the model in Section 1, the marginal product of capital, evaluated at the optimal labor input, is convex in  $s$ .

contribution to expected utility, as demonstrated below.

For use later, we note that,

$$2.8 \quad g(s_t)k_{t-1} = f(k_{t-1}, z^*_t, s_t)$$

the marginal physical product of capital evaluated at the equilibrium labor input times capital equals output.

### Equilibrium

This portion follows Brock (1982, Section 1.5). We start with a definition,

$$2.9 \quad W_t = f(k_{t-1}, z^*_t, s_t) + k_{t-1} \\ = \{g(s_t) + 1\}k_{t-1} = c_t + k_t.$$

The definition simply rearranges the consumption constraint 2.3, evaluated at the equilibrium quantity of labor. Now, conjecture a solution,

$$2.10(a) \quad c_t = \{1-D\}W_t$$

$$(b) \quad k_t = DW_t$$

The conjecture amounts to the life-cycle consumption hypothesis.  $W_t$  is the current value of assets plus dividend and labor income. The owners of capital receive,  $f_k k_{t-1}$ , in dividends; and labor income is,  $f_z z_t$ . Dividend payments plus payments to labor exhaust output,  $f(\dots) = f_k k + f_z z$ , so  $W$  is current income plus accumulated asset holdings. Call  $W$  wealth.

The conjectured life-cycle solution is that the household consumes a constant fraction of the wealth,  $1-D$ , and saves

the remaining fraction,  $D$ . Notice that if the conjecture is correct, then an increase in risk has no effect on current investment, since consumption and capital are constant fractions of wealth. The increased magnitude of the shocks makes the time path of consumption and investment more variable, but since the ratio of capital to wealth is constant in each period, a mean-preserving spread in the distribution of the state of nature has no effect on capital accumulation in the current period.

To verify the solution we substitute 2.10a into the equilibrium condition for capital accumulation 2.6, <7>

$$2.11 \quad 1/c_t = DE_t [\{g(s_{t+1})+1\}/c_{t+1}]$$

$$1/\{1-D\}W_t = DE_t [\{g(s_{t+1})+1\}/\{1-D\}W_{t+1}]$$

and multiply by  $k_t$ , giving,

$$2.11' \quad k_t/\{1-D\}W_t = DE_t [W_{t+1}/\{1-D\}W_{t+1}] = D/\{1-D\},$$

so  $k_t = DW_t$ , confirming the conjecture.

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7 The existence of an equilibrium requires some technical conditions. See Brock (1982) for a more general and rigorous technical discussion. A well-posed maximization problem requires a bounded objective function. So the transversality condition,  $\lim(D^j E_t [U_{c,t+j} \{f_{k,t-i+j+1}\}]) = 0$ , as  $j$  goes to infinity, must hold. The transversality condition holds in this model. Also, the expectation in equation 2.11 must exist, so assume,  $\{g(s)+1\} > 0$ , for all  $s$ . This says that equilibrium output and capital are in the positive orthant for any realization of  $s$ .

### Many Production Processes

The model easily extends to many production processes, and yields the same basic result that an increase in exogenous risk has no effect on current aggregate investment.

Intuitively, the result depends on the life-cycle condition that consumption is proportional to current wealth, and introducing many production processes doesn't alter the basic condition. Many production processes allow the household to diversify risk by investing in many technologies, so an increase in risk does affect the allocation of capital among technologies, but doesn't affect aggregate investment.

Assume  $N$  technologies of the form in the consumption constraint 2.3,

$$2.12 \quad c(i)_t = f(i, k(i)_{t-1}, z(i)_t, s_t) - \{k(i)_t - k(i)_{t-1}\}$$

where  $c(i)/C$  is the share of aggregate consumption produced by the  $i^{\text{th}}$  technology, and  $k(i)/K$  is the share of aggregate capital allocated to the  $i^{\text{th}}$  technology. Let  $C$  and  $K$  denote the aggregate capital stock and aggregate consumption. And define  $W_t = \sum_j \{f(j, k(j)_{t-1}, z(j)_t, s_t) + k(j)_{t-1}\}$  as aggregate wealth.

Using the fact that the marginal physical product of capital, evaluated at the equilibrium labor input, is

independent of capital, we can write the capital accumulation equation for each technology as,

$$2.13 \quad 1/C_t = DE_t[\{g(i, s_{t+1})+1\}/C_{t+1}]$$

Now, conjecture that solution 2.10 holds for the aggregates. Multiplying by  $k(i)_t$ , substituting the conjecture in 2.10(a) for consumption, and rearranging gives,

$$2.14 \quad k(i)_t = E_t[\{g(i, s_{t+1})+1\}k(i)_t/W_{t+1}]DW_t \\ = E_t[w(i)_{t+1}/W_{t+1}]DW_t$$

where,  $w(i)_{t+1} = f(i, \dots)_{t+1} + k(i)_t$ . Equation 2.14 says that capital in the  $i^{\text{th}}$  technology is a fraction of aggregate capital, since  $DW_t$  equals  $K_t$ , by conjecture 2.10(b). The fraction,  $E_t[w(i)_{t+1}/W_{t+1}]$ , is the expected share of next period's wealth generated by the  $i^{\text{th}}$  technology.<sup>8</sup> Since the sum of the fractions equals one,

$$2.15 \quad \sum E_t[w(i)_{t+1}/W_{t+1}] = E_t[dw(i)_{t+1}/W_{t+1}]$$

the conjectured solution is verified.

Now, an increase in exogenous risk--say a mean-preserving spread in the distribution of  $s_{t+1}$ -- will not affect aggregate investment, but the shares of capital allocated to the sectors will adjust to optimally diversify risk. Define,

$$2.16 \quad \theta(i)_t = (k(i)/K)_t$$

<sup>8</sup> An interior solution requires that all the fractions are positive. Constrained solutions also exist, where stochastically dominated technologies get no capital, eg if  $k(i)/K < 0$ , set  $k(i)/K = 0$ .

as the share of capital in the  $i^{\text{th}}$  technology. Substituting  $i$  into the definition of aggregate wealth gives,

$$2.17 \quad W_{t+1} = \left\{ \sum_j g(j, s_{t+1}) i(j)_{t+1} \right\} K_t.$$

So we can write the shares of capital in each technology as,

$$2.18 \quad \theta(i)_t = E_t \left[ \frac{\{g(i, s_{t+1}) + 1\} i(i)_t}{\left\{ \sum_j g(j, s_{t+1}) \theta(j)_{t+1} \right\}} \right]$$

a set of  $N$  nonlinear equations in  $N-1$  unknowns (the fractions sum to one).

The capital share equations 2.18 are roughly analogous to the covariance matrix of portfolio payoffs in the traditional capital-asset pricing model. The covariance matrix of portfolio payoffs depends on the distribution of random security returns times the security weights (number of shares in the portfolio). The shares of capital in 2.18, depend on the random marginal product of capital in the  $i^{\text{th}}$  technology,  $g(i, s)$ , the relationship between the marginal product of capital in the  $i^{\text{th}}$  technology and the marginal products of capital in all other technologies, and the shares of capital (weights) allocated to each technology,  $\theta(j)$ . A change in the distribution of  $s$  changes the joint distribution of the marginal products of capital, say  $G(g(1, s), g(2, s) \dots)$ , and as society reallocates its portfolio of capital among technologies the weights,  $\theta(j)$ , on the random payoffs also change. The simultaneous solution to 2.17 gives the optimal allocation of capital shares.



Equation 2.10b,  $K_t = DW_t$ , gives the aggregate capital allocation.

### Summary

This paper examines the effect of risk on the firm's demand for capital and the effect of risk on the equilibrium allocation of capital. Capital is an asset the firm uses to transfer sales between periods, and an asset society uses to transfer consumption between periods.

The firm values an additional unit of capital in terms of its contribution to expected revenue. Capital's contribution depends on the realizations of the random variables and the factor inputs. The firm diversifies risk through the mix of inputs. For example, the firm varies its labor input with the price of output, and additional labor increases the marginal product of capital. So, an increase in price risk can increase capital's contribution to expected revenue. If the marginal revenue product of capital is convex in a random variable, then a mean-preserving spread in that variable increases capital's contribution to expected revenue. Convexity, or concavity, in the marginal revenue product of capital is roughly analogous to the asset's beta in the traditional capital-asset pricing model. If the marginal revenue product of capital is convex in a random variable, then an increase in the variance of that variable increases the asset's value, as it would for a negative beta asset.

The value of an additional unit of capital to society depends on its contribution to expected utility. Commodity, factor, and asset prices, which are random exogenous variables to individual agents, are endogenous variables in a general equilibrium. The random state of nature is exogenous to the economy. Society diversifies risk by allocating capital among the risky production processes. This paper presents a simple dynamic general equilibrium model to analyze the effect of risk on the equilibrium allocation of capital. The system of share equations for capital allocation is roughly analogous to the covariance matrix of portfolio payoffs in the traditional capital-asset pricing model. In the dynamic general equilibrium model an increase in exogenous--state of nature--risk causes a reallocation of capital among the production technologies, but no increase in aggregate investment. The increase in risk makes consumption and investment more variable, but it does not change the intertemporal allocation of resources. This fits my intuition that an increase in nondiversifiable risk should not cause society to sacrifice additional current consumption for riskier future consumption.

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### Appendix: Adjustment Costs

Abel, Hartman, and Pindyck use models of the firm, with linearly-homogeneous production functions and a cost to adjusting the capital stock, to examine the effect of risk on the firm's optimal investment decision. This appendix adds a cost to adjusting capital to the model in Section 1. Adjustment costs drive a wedge between the market value of a unit of capital and the internal value to the firm. Nevertheless, the results parallel the results in Section 1; if the marginal revenue product of capital is convex in a random variable, then a mean-preserving spread in the random variable increases the firm's demand for investment.

#### The Model

The model of the firm, the definition of the variables, and the stochastic specification, are the same as Section 1, except we add a cost to adjusting capital,

$$A1 \quad q_t = f(k_{t-1}, z_t, s_t) - (P/p)_t I_t - h(I_t) - (w/p)_t z_t,$$

$$\text{where } I_t = k_t - k_{t-1}.$$

The adjustment cost function,  $h(I)$ , is increasing in  $I$ , and zero at  $I=0$ .

Each period the firm invests until,

$$A2 \quad P_t + h_1(I_t) = RE_t [p_{t+1} f_{k,t} + \{P_{t+1} + h_1(I_{t+1})\}],$$

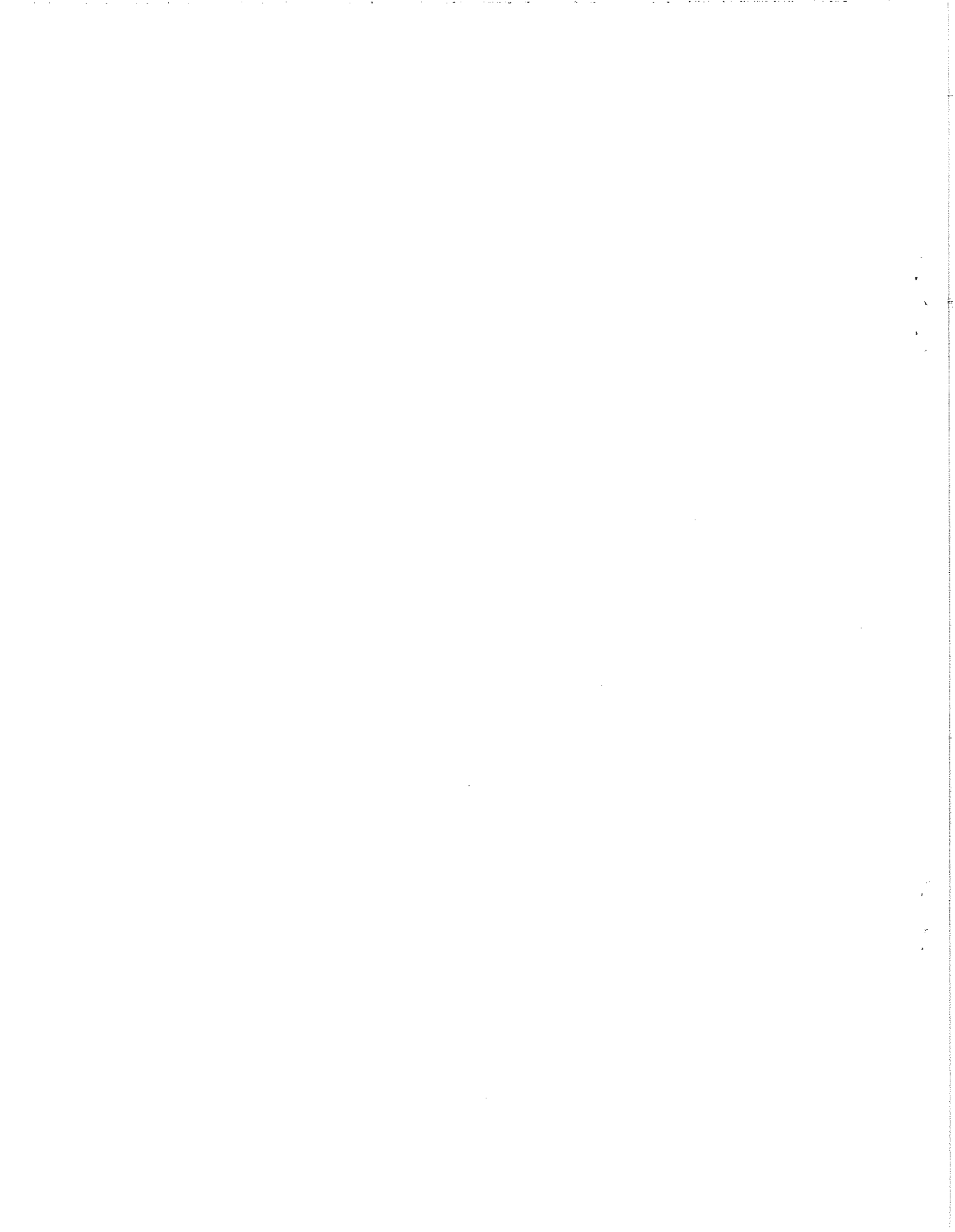
the discounted value of the expected marginal revenue product of capital plus the expected market value of a unit of capital adjusted for the marginal cost of reducing the capital stock, equals the current market price of a unit of capital plus the marginal cost of adding capital. The internal cost to adjusting capital drives a wedge between the market price and the value to the firm; otherwise, equation A2 is the same as equation 1.7.

Since the transition equation A2 holds for any adjacent periods, recursive substitution for,  $P_{t+j} + h_1(I_{t+j})$ ,  $j=1, 2, \dots$ , gives,

$$A3 \quad P_{t+h}(I_t) = \sum_{j=1} R^j E_t [p_{t+j} f_{k,t-1+j}].$$

At a maximum, the present value of the expected marginal revenue product of capital stream equals the market cost plus the internal marginal cost of adding a unit of capital.

If the marginal revenue product of capital is convex in a random variable, then a mean-preserving spread in the random variable increases the expected present value on the right-hand side of A3 and the firm's demand for capital.



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