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Brand or Variety Choices and Periodic Sales as Substitute Instruments for Monopoly Price Discrimination

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Abstract We study the puzzle that sellers often employ diverse strategies in terms of carrying multiple brands and holding periodic sales. These two selling tools can be substitute instruments to induce consumer self selection and implement price discrimination. We analyze the factors that affect a seller's choice between the two pricing instruments and show how different combinations of the two instruments can be optimal under alternative market conditions. A seller may, surprisingly, increase her total number of offers when it becomes more costly to carry brands or hold sales if there are decreasing marginal costs of the alternative selling tool.

Keywords Brands · Multiple varieties · Price discrimination · Sales · Self selection

1 Introduction

Carrying multiple brands, varieties, or models in a product category and holding periodic sales for some of them are two common marketing strategies for sellers with market power. In the presence of consumers with heterogeneous demands for the product either strategy can be used to induce consumer self-selection and implement second-degree price discrimination (Tirole 1988; Salant 1989). Sellers often employ diverse strategies in terms of using these two tools. For example, some retailers use

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extensive sales and specials, often called "high-low pricing", while others maintain stable prices with few or no sales, often called "everyday low pricing" (Lal and Rao 1997).

Why do sellers adopt such widely divergent strategies? We argue that contemporaneous price discrimination through offering multiple varieties or brands at differentiated prices and intertemporal price discrimination through holding periodic sales of a particular brand/variety are substitute marketing strategies for a seller to induce consumer self selection and implement second-degree price discrimination. Given the same discounted price, consumers are indifferent between consuming a lower-quality brand/variety now and consuming a higher-quality brand/variety later, as long as the two alternatives provide the same present value of utility.

This substitutability allows a seller to use different combinations of carrying brands/varieties and holding sales to induce consumer self selection, with the specific optimal choice determined by the relative costs of offering multiple brands versus holding sales and various characteristics of the seller's products and customers. We develop a model to explore this intuition. We derive the seller's optimal total number of offers (each defined as a product price and quality level, and the time when that price-quality pair is available) and the distribution of those offers between number of brands/varieties provided at regular prices and number of promotions from putting some of those brands/varieties on sale at a later time.

Analysis of the equilibrium provides insight into the factors that affect a seller's choice between contemporaneous and intertemporal price discrimination and helps explain sellers' diverse marketing strategies by showing how different combinations of the two marketing instruments can be optimal under alternative market conditions. A no-sales strategy is likely for products with a narrow quality range, relatively homogeneous consumers, and high sales costs relative to brand/variety carrying costs. A wider quality range and greater consumer heterogeneity cause a seller to provide more offers and be more likely to use both multiple brands/varieties and periodic sales. Interestingly, a seller may increase its number of offers when either carrying brands/varieties or holding sales becomes more costly if there is decreasing marginal cost of the alternative selling tool.

We discuss the relevant literature in Sect. 2 of the paper and then set forth the model in Sect. 3. Section 4 develops the model's equilibrium and derives comparative static results, while Sect. 5 concludes.

2 Quality and Intertemporal Price Discrimination

Prior studies have mostly focused on contemporaneous or intertemporal price discrimination in isolation with little consideration given to the choice of instrument(s) to use. Mussa and Rosen (1978) showed that a monopolist can provide different varieties of a basic product to implement second-degree price discrimination among consumers who put heterogeneous valuations on different varieties of the good.[1] Consumers whose preferences for the product are most intense can be induced to purchase

[1] P'ng and Lehman (2007) call this marketing strategy "indirect segment discrimination".

high-quality and high-price versions of the product, while consumers with milder preferences choose lower quality, cheaper versions. A monopolist can, thus, select a menu of quality-price pairs to maximize profits subject to the constraints that the consumers are willing to purchase the brand/variety targeted to them (individual rationality) and are not tempted to purchase a brand/variety targeted to a different consumer segment (self selection).

Maskin and Riley (1984) extended this work to the case when consumers' preferences are different over both quality and quantity of a good, while Gabszewicz et al. (1986) studied a monopolist's optimal range in terms of product quality and found that the monopolist either provides the maximum number of varieties possible or offers only one variety, depending on how much tastes differ among consumers. Donnenfeld and White (1988) further analyzed the scenario where consumers are heterogeneous in both absolute and marginal willingness to pay for quality and showed that a monopolist may distort or restrict the quality range of product at either the low end or the high end.

Even if consumers discount future consumption at the same rate, the forgone utility from deferring consumption is greater for consumers with the highest intensity of preference because their potential utility from consumption is greatest. These consumers may be persuaded to purchase the product at a high initial price, while those with milder preferences wait to purchase it on sale. Thus, intertemporal price variations for a product may also be a device to induce consumer self selection and implement price discrimination.

The literature on intertemporal price strategies and sales has focused on the conditions when intertemporal price variations can be a tool for self selection and second-degree price discrimination. Such discrimination is not possible if a seller is unable to commit to a dynamic price strategy. Because rational consumers anticipate the monopolist's incentive to engage in successive price reductions over time to capture the patronage of lower-valuation consumers, no one is willing to buy at prices above marginal cost. This basic intuition, due to Coase (1972), was confirmed in formal models under varying conditions by Stokey (1981), Gul et al. (1986), and Sobel (1991).

However, many other studies have examined commitment devices that enable a seller to avoid this outcome. Tirole (1988) and Waldman (2003) provide summaries of this literature. Examples include fixed capacity (Bulow 1982), increasing marginal cost (Kahn 1986), reputation formation (Ausubel and Deneckere 1989), and contractual provisions such as most-favored-customer and best-price clauses (Butz 1990). More recently Deneckere and Liang (2008) showed that finite durability of the good (planned obsolescence) prevents emergence of Coase's equilibrium.

Even with commitment power, intertemporal price discrimination is not necessarily optimal for a monopolist. Stokey (1979) assumed that a monopolist had commitment power over a finite horizon and faced a continuum of consumers distinguished by differing reservation prices. She showed that price discrimination did not emerge in the standard case—all sales take place when the product is first introduced.

Conlisk et al. (1984) and Sobel (1984), however, showed that sellers hold periodic sales to conduct intertemporal price discrimination when some customers have high valuations and are impatient relative to others, and new consumers enter the market in each period. The entry of new customers provides a commitment device because it is rational for the monopolist to sell to only high-value entrants until enough low-value customers have accumulated to warrant holding a sale.

Landsberger and Meilijson (1985) showed that intertemporal price discrimination is optimal for a seller with commitment who has a lower discount rate than do consumers, and Van Cayseele (1991) found that intertemporal price discrimination is more profitable for a monopoly seller than is a single-price strategy when the seller has commitment power and high-valuation consumers run the risk of being rationed if they defer consumption in hope of obtaining a lower price.

Few studies have examined brand/variety choices and dynamic price strategies jointly, and none has established the inherent substitutability between quality/variety and intertemporal instruments of price discrimination that we demonstrate here. Salant (1989) showed that the price-quality choice problem of Mussa and Rosen (1978), the intertemporal pricing problem of Stokey (1979), and the price-quantity choice problem of Spence (1977) could all be cast in a single, unified framework that illuminates the conditions when self selection and second-degree discrimination are profitable for a monopolist, but he gave no consideration to the choice of which instrument to use.

Bagnoli et al. (1995) showed that a monopolist may achieve higher profits by offering a sequence of quality-price menus of multiple varieties over time than can be obtained from a single menu. Kühn (1998) considered a durable-good monopolist, who sells a high-quality and low-quality version (subject to failure) of a product, which he calls, respectively, a "durable" and "nondurable" good. He showed that the monopolist is able to conduct intertemporal price discrimination in this setting because the possibility of selling the nondurable provides a commitment device for the seller who when selling the durable good must be compensated for the forgone opportunity to sell the nondurable version of the good. Kumar (2002) examined a model with customer resales where a monopolist could vary both price and quality over an infinite horizon. However, only one quality can be offered at a given time in his model.

3 The Model

Inducing consumer self selection enables the seller to segment consumers based upon differences in intensity of preference or willingness to pay for the seller's product. As noted, both contemporary price discrimination through carrying multiple brands or varieties and intertemporal price discrimination through holding periodic sales can induce consumer self selection.

However, inducing self selection comes at a cost. First, the availability of offers targeted to consumers with lower valuations for the product limits the seller's ability to extract surplus from consumers with high valuation because they may choose an offer targeted for low-valuation consumers. Second, to relax this self-selection constraint for high-valuation consumers, the seller has an incentive to distort the quality of offers targeted to low-valuation consumers, thereby creating a deadweight loss. Salant (1989) derives general conditions under which inducing self selection is optimal for a seller in the presence of these considerations.

Another cost of inducing self selection, although one largely ignored in the literature, is the cost associated with making multiple offers of the basic product. There will normally be additional costs associated with offering multiple brands or varieties instead of just one and also costs associated with holding periodic sales. Depending upon the circumstances of a seller, these brand/variety carrying costs and sales costs may differ considerably and affect the seller's decision as to how many offers to make and their distribution between brands/varieties and periodic sales.

We extend the Gabszewicz et al. (1986) model to provide a framework that permits the joint analysis of contemporaneous and intertemporal price discrimination. Given our interest in studying contemporaneous and intertemporal price discrimination in those settings when it is optimal for a seller to induce self selection, we make a formal assumption on the range of consumer heterogeneity or "income" relative to the product quality range to insure that self selection is induced in equilibrium.

3.1 The Seller

We consider a monopoly that can sell multiple varieties, brands, or models of a product line to consumers and also put some of them on sale. The seller could be a retailer who chooses among brands of a product offered by competitive manufacturers, sets prices for the brands, and makes decisions on whether to hold sales for some of the brands.

The model also applies to a monopoly manufacturer who produces and sells multiple varieties or models of a product line to consumers through competitive retailers. The manufacturer could be a monopolist per se or a seller of a highly differentiated product line with strong brand identity so that the customer base is primarily those who have committed ex ante to purchase a product in the manufacturer's line. To simplify the analysis, we ignore any interactions between demand for this product line and demand for any other goods sold by the firm.

The seller faces heterogeneous consumers who are in the market to purchase at most one unit of the product over a finite horizon specified as [0, T]. The seller chooses a marketing strategy at the outset of the horizon in terms of price, quality, and timing of offers for the product line to maximize profits over the horizon. Planning over such a finite horizon is consistent with actual business practices and is known in the marketing literature as the "trade promotion calendar" (Silva-Risso et al. 1999; Besanko et al. 2005).[2]

Consistent with recent contributions such as Koh (2006) and Board (2008) and observed behavior (Waldman 2003; Zhang and Krishnamurthi 2004), we assume that the seller is able to commit to this strategy. Our results, thus, establish a benchmark for the outcomes that a seller can achieve when it can commit (Board 2008).

[2] In the case of manufacturer-set promotions, retailers generally are provided with the trade promotion prices, distribution of promotions across brands, and dates in advance (Hall et al. 2003; Besanko et al. 2005). Manufacturers normally do not deviate from the trade promotion policies stated in the calendar (Zhang and Krishnamurthi 2004).

The number of vertically differentiated brands or varieties of the product that are available is exogenous and denoted by J.[3] (Although the context need not be brands per se, for parsimony of presentation we henceforth refer to the J products as "brands".) The qualities that consumers uniformly associate with these brands are $u_1, u_2, \ldots, u_J$, where $0 < \underline{u} = u_1 < u_2 < \cdots < u_J = \bar{u}$.

These qualities may be objective or "perceived" in the sense that they exist in consumers' minds due to the effects of brands and advertising. The seller's problem is to choose the total number of offers, N, to make and the distribution of those offers between $M \in [1, J]$ "regular" offers from carrying M brands and charging a regular (nonsale) price for them,[4] and $H = N - M$ promotional offers created by lowering the (discounted) prices of some of the M brands at some later period(s) during the time horizon. These H promotional offers may include multiple discounts at different points in time for a given brand.

We consider the model framework to apply to a rather broad range of semi-durable goods that feature multiple brands, varieties, or models, are purchased periodically by consumers, and are unlikely to be inventoried by them. For example, a car, appliance, or consumer electronics manufacturer (or retailer) knows that a set of consumers are in the market possibly to purchase a product from its line over a given horizon, and each will accept the offer that maximizes his utility or not purchase at all if no satisfactory offer is forthcoming.

In addition, the model may also offer insights regarding retailing of some nondurable consumer goods or services that are nonstorable and subject to periodic purchases. For example, shoppers may adopt the rule of serving an expensive food item like steak periodically, if a suitable deal is presented. A retailer can induce self selection by offering multiple grades and cuts and/or by offering periodic sales of some items.

Each seller offer thus consists of the price (P) and quality level (u) of the product, and time (t) when the price-quality pair is available. Regular offers (subscript r) are characterized as the triple $\left(P_{r,m}, u_{r,m}, t_{r,m}\right)$, $m = 1, 2, \ldots, M$. Because consumers are assumed to be impatient, anyone who accepts a regular offer will do so at the outset of the horizon, so $t_{r,m} = 0$. The H promotional offers (subscript h) that arise from putting some of the M brands on sale are characterized as $\left(P_{h,n}, u_{h,n}, t_{h,n}\right)$, where $n = M + 1, M + 2, \ldots, N$. By definition, consumers must wait some time for a brand to go on sale, so $0 < t_{h,n} \leq T$.

We assume that unit variable procurement cost is the same for all quality levels, and for simplicity set those costs equal to zero. This assumption is common in studies of vertical differentiation as a way to simplify the analysis.[5] In our model this assumption

[3] In the context of a retailer, J is the total brands available from competitive manufacturers. In the case of a manufacturer, J is the total number of varieties or models that are feasible to produce and market in the short run, e.g., due to technological constraints.

[4] This seller decision can reflect a manufacturer's choice of M brands to produce among J brands that are technologically feasible, or the decision to offer M among J extant brands in a particular market. For example, auto manufacturers usually carry different models and different numbers of models in Europe relative to the US.

[5] A partial listing of prior authors who have utilized this assumption includes Shaked and Sutton (1982), Gabszewicz et al. (1986), Bolton and Bonanno (1988), Bental and Spiegel (1995), Bonnisseau and Lahmandi-Ayed (2006), and Jing (2007).

enables a recursive approach to be utilized to solve the model and obtain analytical solutions. As we discuss subsequently, the paper's core results continue to hold when the unit variable procurement cost is increasing in the quality level.

The seller incurs a carrying cost, $B(M, b)$, for stocking M brands of the product and a sales cost, $S(H, s)$, for holding H sales, where $b > 0$ $(s > 0)$ is a cost-shift parameter for carrying brands (holding sales). The marginal cost of carrying brands and holding sales is $B_1(M, b) = \partial B(M, b)/\partial M$ and $S_1(H, s) = \partial S(H, s)/\partial H$, respectively.[6] Each marginal cost function is increasing in its cost-shift parameter: $\partial B_1/\partial b > 0$, $\partial S_1/\partial s > 0$. We assume that B_1 (S_1) is monotonically changing in the number of brands (sales) and define $B_{11} = \partial^2 B(M, b)/\partial M^2$ and $S_{11} = \partial^2 S(H, b)/\partial H^2$. The total cost of providing M regular offers and $H = N - M$ promotional offers is thus $TC(N, M) = B(M, b) + S(N - M, s)$.

3.2 Consumer Behavior

Specification of consumers' utility follows Gabszewicz et al. (1986). A given mass of consumers is in the market to buy a version of the product during the marketing horizon, but they are heterogeneous in terms of their willingness and/or ability to purchase it. We denote this consumer heterogeneity by θ, which can be interpreted as the income that a consumer has available to spend over the horizon (Shaked and Sutton 1982). We assume that θ is distributed uniformly in the range $[\theta_0, \lambda\, \theta_0]$, with $\theta_0 > 0$, $\lambda > 1$, and density function $f(\theta) = 1/(\lambda\, \theta_0 - \theta_0)$, where λ measures the degree of heterogeneity among consumers, and θ_0 indicates the "base" income level of consumers.

Consumers are assumed to be forward looking and can anticipate all N offers that will be made during the entire time horizon.[7] Each consumer purchases either one unit of the good or none during the horizon. A consumer will accept the offer that yields the highest utility, if this utility is higher than his reservation utility, $\underline{U}(\theta) = \theta\, u_0$, where $0 < u_0 < \underline{u}$; otherwise he will not buy at all. By accepting an offer, (P, u, t), a consumer with income θ pays the price P and obtains one unit of product with quality u at time t. His discounted utility is defined as $U(\theta, P, u, t) = \left(\theta - Pe^{-\gamma t}\right) u\, \delta(t) = \left(\theta - Pe^{-\gamma t}\right) ue^{-\rho t}$, where $\gamma > 0$ is the interest rate, and $\delta(t) = e^{-\rho t}$ is the quality discount factor, $\rho > 0$. The lowest value of $\delta(t)$ is $\delta(T) = e^{-\rho T} = \tilde{\delta} \in (0, 1)$.

Consumers with higher θ are willing to pay more to purchase the product than are their lower-income counterparts. The quality discount factor indicates that all consumers prefer consuming earlier than later, but the intensity of time preference is an increasing function of θ. We define a new variable, $v = u\, \delta(t)$, to represent the discounted value of the quality of an offer, which we call the "present quality". We also

[6] For example, a retailer's brand carrying costs would include the transactions costs of dealing with multiple brand manufacturers and stocking multiple brands. Retailers' sales costs would include the array of activities collectively known as "menu costs", and possible promotional costs due to notifying consumers of the sale price.

[7] This assumption is also standard in the literature on durable-goods monopoly (e.g., Conlisk et al. 1984; Sobel 1991; Kühn 1998) and is supported broadly by research on consumer behavior [see especially Krishna (1989) and the survey by Chintagunta et al. (2006)].

discount the prices of all offers to the current price level at the beginning of the time horizon ($t = 0$) by defining $p = Pe^{-\gamma t}$.

The introduction of the new variables v and p transforms an offer from a price-quality-time bundle, (P, u, t), to a discounted price and present quality bundle, (p, v). We order the N present qualities, $v_{r,1}, v_{r,2}, \ldots, v_{r,M}, v_{h,M+1}, v_{h,M+2}, \ldots, v_{h,N}$, and re-label them as $v_k, k = 1, 2, \ldots, N$, where $v_1 < v_2 < \cdots < v_N$. Correspondingly, we re-label the N regular and promotional offers as $(p_1, v_1), (p_2, v_2), \ldots, (p_N, v_N)$. The seller thus provides N discounted price-present quality bundles, (p, v), to consumers through her choices of M regular offers and N − M promotional offers. Accordingly, a consumer's utility from accepting an offer (p_k, v_k) can be restated as:[8]

$$\tilde{U}(\theta, p_k, v_k) = (\theta - p_k)\, v_k = U(\theta, P_k, u_k, t_k) = \left(\theta - P_k e^{-\gamma t_k}\right) u_k\, \delta(t_k).$$

Consumers only care about the present quality of an offer regardless of whether it comes from a regular offer or a promotional offer. Given the same discounted price, a consumer is indifferent between consuming a low-quality brand in the current period and a high-quality brand at a later period, if each provides the same present quality, v.

4 Equilibrium Analysis

Define a feasible quality range of the product as $[u_0, \bar{u}]$ such that (i) consumers are willing to pay a nonnegative price to consume a product with any quality in this range, and (ii) all qualities of the range can be produced using current technology. The present qualities of the M regular offers must be chosen from the quality set of the available brands. That is, $u_{r,m} = v^{\varepsilon}_{r,m}\{u_1, u_2, \ldots, u_J\}$, where $m = 1, 2, \ldots, M$. We assume that there are a sufficiently large number of available brands J and that their qualities are relatively evenly distributed over the quality range $[u_0, \bar{u}]$ to enable this condition to hold and, also, ensure that carrying some or all brands plus putting some or all brands on sale can provide all product (present) quality values in the range $[u_0, \bar{u}]$.[9]

The seller chooses a marketing strategy to maximize her discounted profit, which is equal to her discounted revenue minus her total selling costs. We denote the income of consumers who are indifferent between accepting two adjacent offers, (p_{k-1}, v_{k-1}) and (p_k, v_k), as $\theta_k = (p_k v_k - p_{k-1} v_{k-1})/(v_k - v_{k-1})$ with $k \geq 2$. Similarly, the income of consumers who are indifferent between accepting (p_1, v_1) and not buying

[8] A literal interpretation of our version of the Gabszewicz et al. utility function is that a consumer has θ units of income available at the outset of the horizon. If a consumer purchases the product, he immediately spends $\theta - p$ units on the numeraire commodity and saves p units at interest rate γ in anticipation of purchasing the product for price P in period t.

[9] This assumption works well when the seller is a manufacturer, who can choose varieties or models with qualities along a continuum. That means the quality set of the available varieties for the manufacturer is the quality range, $[u_0 \bar{u}]$, so the qualities of the M regular offers can always be chosen from the quality set of the available brands and carrying the M brands plus putting some or all of them on sale can provide all product quality values in the range $[u_0, \bar{u}]$. When the seller is a retailer who chooses brands from the available manufacturers' brands, store-owned (private-label) brands can represent a device for retailers to fill in any "missing" brands that are needed to provide the M regular offers. We consider implications of relaxing this assumption in Appendix 2 of our working paper (Xia and Sexton 2007).

the product at all is $\theta_1 = p_1 v_1/(v_1 - u_0)$. We assume that this type of consumer exists, which requires that $\theta_0 < \theta_1$ at the equilibrium. In turn this inequality implies a condition on the distribution of consumer income, λ, relative to the width of the quality range, $\bar{u}/u_0$:

$$\lambda > N^* + 1 - \left(N^* - 1\right)/\left(\bar{u}/u_0\right)^{1/N^*},$$

where N^* is the value of N at the equilibrium as defined in Eq. (11) below, and it is a function of the model's parameters, θ_0, λ, u_0, $\bar{u}$, b, s, and $\tilde{\delta}$.

Consumers with $\theta \in [\theta_0, \theta_1)$ do not buy any product; consumers with $\theta \in [\theta_k, \theta_{k+1})$ with $1 \leq k < N$ accept the offer (p_k, v_k); and consumers with $\theta \in [\theta_N, \lambda\,\theta_0]$ accept the offer (p_N, v_N). Thus, the discounted revenue function is

$$R(N, \mathbf{p}, \mathbf{v}) = \left[p_1(\theta_2 - \theta_1) + p_2(\theta_3 - \theta_2) + \cdots + p_N\left(\lambda\theta_0 - \theta_N\right)\right]\left[1/\left(\lambda\theta_0 - \theta_0\right)\right],$$

where, $\theta_1 = p_1 v_1/(v_1 - u_0)$ and $\theta_k = (p_k v_k - p_{k-1} v_{k-1})/(v_k - v_{k-1})$ for $k = 2, 3, \ldots, N$.

The profit function is

$$\pi(N, \mathbf{p}, \mathbf{v}, M) = R(N, \mathbf{p}, \mathbf{v}) - TC(N,M) = R(N, \mathbf{p}, \mathbf{v}) - [B(M, b) + S(N - M, s)].$$

Note that the only variable to appear in both the revenue and the selling-cost functions is N. The variables p_k and v_k are in the revenue function only, and the variable M is in the cost function only. Thus, the seller decides the optimal marketing strategy in two stages: In the first stage, based on consumers' rational expectations about the values of p_k, v_k, and M, the seller chooses the optimal number of offers to maximize her profit. In the second stage, given N, the seller (i) sets the discounted prices, p_k, and present qualities, v_k, of the offers to maximize discounted revenue, and (ii) chooses the optimal number of brands, M, and sales, $H = N - M$, to minimize the total selling cost of providing N offers. The model is solved using backward induction.

4.1 Revenue Maximization with Given N

In the second stage, the seller's revenue-maximization problem given N offers is

$$\max_{\{\mathbf{p},\mathbf{v}\}} R(\mathbf{p}, \mathbf{v}|N) = \left[p_1\left(\theta_2 - \theta_1\right) + p_2\left(\theta_3 - \theta_2\right) + \cdots + p_N\left(\lambda\theta_0 - \theta_N\right)\right]\left[1/\left(\lambda\theta_0 - \theta_0\right)\right],$$

$$\text{s.t. } u_0 \leq v_1 \leq v_2 \leq \cdots \leq v_N \leq \bar{u}, 0 \leq p_1 \leq p_2 \leq \cdots \leq p_N,\ \theta_1 = p_1 v_1/(v_1 - u_0),\ \text{and}$$
$$\theta_k = \left(p_k v_k - p_{k-1} v_{k-1}\right)/\left(v_k - v_{k-1}\right) \quad \text{for } k = 1, 2, \ldots, N.$$

From the first-order conditions (FOCs) for p_1 through p_N, we obtain

$$(p_2 - p_1)\left[(v_2 + v_1)/(v_2 - v_1)\right] = p_1[(v_1 + u_0)/(v_1 - u_0)], \tag{1}$$

$$(p_{k+1} - p_k)\left[(v_{k+1} + v_k)/(v_{k+1} - v_k)\right] = \left(p_k - p_{k-1}\right)\left[\left(v_k + v_{k-1}\right)/\left(v_k - v_{k-1}\right)\right] \quad \text{for} \quad k = 2, \ldots, N-1, \tag{2}$$

$$(p_N - p_{N-1})\left[\left(v_N + v_{N-1}\right)/\left(v_N - v_{N-1}\right)\right] = \lambda\theta_0 - p_N. \tag{3}$$

Because $\partial R/\partial v_N = v_{N-1}(p_N - p_{N-1})^2/\left[(v_N - v_{N-1})^2 (\lambda\theta_0 - \theta_0)\right] > 0$ ($R(\cdot)$ is increasing in the value of the highest present quality), the optimal choice of v_N is $\bar{u}$, the highest available quality (Gabszewicz et al. 1986). From the FOCs for v_1 through v_{N-1}, we obtain

$$(v_1 - u_0)^2 v_2 \Big/ \left[(v_2 - v_1)^2 u_0\right] = p_1^2/(p_2 - p_1)^2, \tag{4}$$

$$(v_k - v_{k-1})^2 v_{k+1} \Big/ \left[(v_{k+1} - v_k)^2 v_{k-1}\right] = (p_k - p_{k-1})^2 \Big/ (p_{k+1} - p_k)^2 \quad \text{for } k = 2, \ldots, N-1. \tag{5}$$

Solving Eqs. (1) through (5) and using the condition that $v_N^* = \bar{u}$, we find the optimal choices of discounted prices and present qualities as follows:

$$v_k^* = u_0 (\bar{u}/u_0)^{k/N} \quad \text{for} \quad k = 1, 2, \ldots, N-1 \quad \text{and}$$

$$p_k^* = k\lambda\theta_0\left[(\bar{u}/u_0)^{1/N} - 1\right] \Big/ \left[(N+1)(\bar{u}/u_0)^{1/N} - N + 1\right] \quad \text{for } k = 1, 2, \ldots, N,$$

where $\bar{u}/u_0$ measures the width of the quality range, and $(\bar{u}/u_0)^{1/N} = v_{k+1}^*/v_k^* > 1$ measures the difference at the optimum between every two adjacent present qualities, v_k^* and v_{k+1}^*. Substituting the optimal prices and present qualities into the revenue function yields the maximum discounted revenue function, given N total offers, as

$$R^*(N\,|\theta_0, \lambda, \bar{u}/u_0) = N\left[\theta_0\lambda^2/(\lambda - 1)\right]\left[(\bar{u}/u_0)^{1/N} - 1\right] \Big/ \left[2(N+1)(\bar{u}/u_0)^{1/N} - 2N + 2\right]. \tag{6}$$

4.2 Cost Minimization with Given N

The seller chooses the distribution of N total offers between number of brands carried, M, and number of sales conducted, $H = N - M$, to minimize total selling cost in the second stage, subject to the constraint that enough brands are carried to provide N total offers. When consumers are patient ($\tilde{\delta}$ is large) and/or the product quality range is wide (a large $\bar{u}/u_0$), carrying only one brand (the brand with highest quality $\bar{u}$) plus putting it on sale may not enable the seller to provide the offers with some lowest optimal present qualities, i.e. $\bar{u}\tilde{\delta} > v_1^*$. In that case, the seller needs to carry additional brands with quality levels lower than $\bar{u}$ to be able to provide all N optimal offers. Specifically, the seller can provide at most $\beta(N) = 1 + \log_{\left(1/(\bar{u}/u_0)^{1/N}\right)} \tilde{\delta} \geq 1$ offers by carrying one brand and putting it on sale. Thus, the minimum number of brands that must be carried to provide all N optimal offers is $\underline{M}(N) = N/\beta(N)$, where $1 \leq \underline{M}(N) \leq N$.[10]

To see this result, note first that carrying one brand with v_k plus putting it on sale can provide optimal offers with present qualities in the range $\left[v_k\tilde{\delta}, v_k\right]$. Because the ratio between the highest and the lowest quality in this range, $v_k/\left(v_k\tilde{\delta}\right) = 1/\tilde{\delta}$, is equal to $\left[(\bar{u}/u_0)^{1/N}\right]^{(\beta-1)}$, where $(\bar{u}/u_0)^{1/N}$ is the quality ratio between two adjacent optimal offers, there are at most $(\beta(N) - 1) + 1 = \beta(N)$ optimal present qualities in the range $\left[v_k\tilde{\delta}, v_k\right]$. Thus, carrying one brand plus putting it on sale can provide at most $\beta(N)$ optimal offers.

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[10] In discussing numbers of brands/varieties and sales, we limit our attention throughout to integer values.

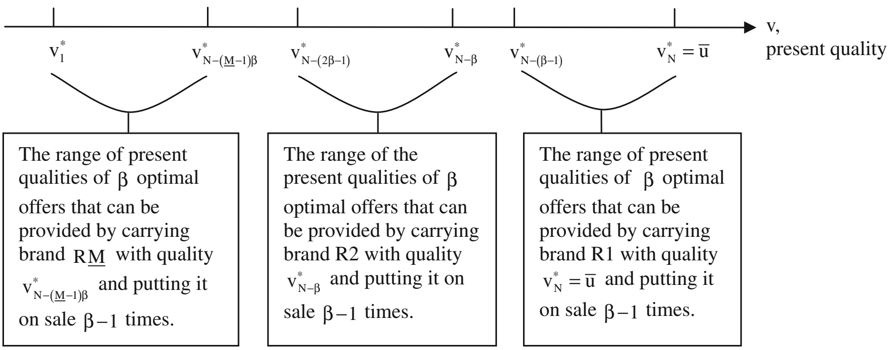

Fig. 1 The general linear product quality space

For example, the seller must carry the brand with the highest quality, $v_N^* = \bar{u}$. Denote this brand as R1. Carrying R1 plus putting it on sale enables the seller to provide offers with the present qualities belonging to $\left[v_N^*\tilde{\delta}, v_N^*\right]$, where $v_{N-\beta}^* < v_N^*\tilde{\delta} = v_{N-(\beta-1)}^*$. Thus, the quality range $\left[v_N^*\tilde{\delta}, v_N^*\right]$ includes the β (N) optimal present qualities: $v_N^*, v_{N-1}^*, \ldots, v_{N-(\beta-1)}^*$. Figure 1 illustrates the general linear product quality space, v, and the range of offers attainable from carrying a brand/quality and offering it on sale.

The formal cost minimization problem is

$$\begin{aligned} &\underset{\{M\}}{\mathrm{Min}}\, TC(M\,|N) = B\,(M, b) + S\,(N - M, s) \\ &\text{s.t.(i)}\quad M \le N, \\ &\quad\text{(ii)}\quad M \ge \underline{M}\,(N)\,. \end{aligned} \tag{7}$$

Define $\hat{M}$ (b,s,N) as the unconstrained solution to the cost-minimization problem.[11] The optimal number of brands is:

$$M^* = \begin{cases} \hat{M}\,(b,s,N) & \text{if}\quad \underline{M}\,(N) \le \hat{M} \le N \quad \text{(Case 1)} \\ N & \text{if}\quad \hat{M} > N \quad \text{(Case 2)} \\ \underline{M}\,(N) & \text{if}\quad \hat{M} < \underline{M}\,(N) \quad \text{(Case 3).} \end{cases} \tag{8}$$

The solutions for M in (8) are substituted back into the objective function to obtain the minimized total cost, TC*(N):

$$TC^*\,(N) = \begin{cases} B\left(\hat{M}, b\right) + S\left(N - \hat{M}, s\right) & \text{if}\quad \underline{M}\,(N) \le \hat{M} \le N \quad \text{(Case 1)} \\ B\,(N, b) & \text{if}\quad \hat{M} > N \quad \text{(Case 2)} \\ B\left(\underline{M}\,(N)\,, b\right) + S\left(N - \underline{M}\,(N)\,, s\right) & \text{if}\quad \hat{M} < \underline{M}\,(N) \quad \text{(Case 3).} \end{cases} \tag{9}$$

[11] The second-order condition to (7) for the unconstrained case is $B_{11}\left(\hat{M}, b\right) + S_{11}\left(N - \hat{M}, s\right) > 0$, and it is assumed to hold. Thus the marginal cost of at least one selling tool is increasing and, if there is decreasing marginal cost for using the other tool, the increasing rate of the marginal cost of the first tool is greater than the decreasing rate of the second tool. When the second order condition does not hold, a monopolist will always choose only one selling tool.

Case 1 occurs when neither constraint to problem (7) binds. The optimal number of brands is $M^* = \hat{M}$ (b,s,N), and the optimal number of sales is $N - \hat{M}$. The number of brands in this case is determined by the seller's relative cost between carrying brands and holding sales and the total number of offers. We obtain $\partial \hat{M}/\partial b < 0$ and $\partial \hat{M}/\partial s > 0$, so the optimal number of brands in the unconstrained case is decreasing in the brand-carrying cost factor and increasing in the sales cost factor.

For given values of market and cost parameters, there are in general multiple choices for the seller (combinations of price, quality, and time) in the unconstrained case to provide the $N - \hat{M}$ optimal promotional offers in terms of which of the $\hat{M}$ brands are offered on sale and the number of sales for a given brand. For example, the choices may range from putting one brand (e.g., the brand with the highest quality, $\overline{u}$) on sale for $N - \hat{M}$ times, to choosing $N - \hat{M}$ different brands and putting each of them on sale once if $N - \hat{M} \leq \hat{M} \Leftrightarrow N \leq 2\hat{M}$.

These choices are equivalent for the seller because cost and profit are the same with any one of the choices—i.e., as long as the seller is able to provide the same present qualities of the $N - \hat{M}$ optimal promotional offers, it doesn't matter whether it is an earlier sales offer of a low-quality brand or a later sales offer of a high-quality brand.

Case 2 occurs when the constraint, $M \leq N$, is binding. In this case, the seller provides all N offers through carrying brands and holds no sales. Case 3 applies when the constraint, $M \geq \underline{M}(N)$, binds. Here the unconstrained optimal number of brands is smaller than the minimum number of brands that are required for the seller to provide all N offers. Thus, the seller's optimal strategy is to carry the required minimum number of brands and hold $N - \underline{M}(N)$ sales—the "minimum-brands-plus-sales" case.

4.3 The Profit-Maximizing Choice of Total Offers, N

Turning now to the first stage, the seller chooses the number of total offers to maximize profit, given the discounted revenue and selling-cost functions:

$$\underset{\{N\}}{\text{Max}}\, \pi\ (N) = R^*(N) - TC^*(N). \tag{10}$$

Assuming an interior solution (i.e., $N^* \geq 1$), the FOC to (10) is $\partial R^*/\partial N - \partial TC^*/\partial N = 0$, where from (6) and (9),

$$\begin{aligned}\partial R^*/\partial N &= MR(N\,|\theta_0, \lambda, \overline{u}/u_0) \\ &= \left[\theta_0\lambda^2/(\lambda-1)\right]\left[(\overline{u}/u_0)^{2/N} - (2/N)\,(\overline{u}/u_0)^{1/N}\ln(\overline{u}/u_0) - 1\right] \Big/ \\ &\quad \left[2\left((N+1)\,(\overline{u}/u_0)^{1/N} - N+1\right)^2\right]\end{aligned}$$

and

$$\partial TC^*/\partial N = \begin{cases} B_1\left(\hat{M}, b\right) = S_1\left(N - \hat{M}, s\right) & \text{if } \underline{M}(N) \leq \hat{M} \leq N \quad \text{(Case 1)} \\ B_1\,(N, b) & \text{if } \hat{M} > N \quad \text{(Case 2)} \\ B_1\left(\underline{M}(N), b\right)/\beta\,(N)^2 & \\ \quad +S_1\left(N - \underline{M}(N), s\right)\left[1 - \left(1/\beta\,(N)^2\right)\right] & \text{if } \hat{M} < \underline{M}(N) \quad \text{(Case 3).} \end{cases}$$

Table 1 The effects of variables/parameters on total number of offers, N

Parameter/variable	Case 1 (unconstrained)	Case 2 (no-sales)	Case 3 (minimum-brands-plus-sales)
b	$-$ if $S_{11} > 0$ 0 if $S_{11} = 0$ $+$ if $S_{11} < 0$	$-$	$-$
s	$-$ if $B_{11} > 0$ 0 if $B_{11} = 0$ $+$ if $B_{11} < 0$	0	$-$
$\bar{u}/u_0$	+	+	?
θ_0	+	+	+
λ	+	+	+
$\tilde{\delta}$	0	0	?

The symbols, +, −, 0, and ? denote positive, negative, no, and indeterminate effect, respectively

In each of the three cases N^* is the solution of an implicit equation relating N to the model's parameters, θ_0, λ, $\bar{u}/u_0$, b, s, and $\tilde{\delta}$. Denote the optimal numbers of offers for the three cases as N_1^*, N_2^*, and N_3^*, respectively. For case 1 (unconstrained), N_1^* is the solution of the equation,

$$0 = F_1(N) \stackrel{\text{def}}{=} MR(N\,|\theta_0, \lambda, \bar{u}/u_0) - B_1\left(\hat{M}, b\right).$$

For case 2 (no sales), N_2^* is the solution of the equation,

$$0 = F_2(N) \stackrel{\text{def}}{=} MR(N\,|\theta_0, \lambda, \bar{u}/u_0) - B_1(N, b).$$

For case 3 (minimum brands plus sales), $N_3^*$is the solution of the equation,

$$0 = F_3(N) \stackrel{\text{def}}{=} MR(N\,|\theta_0, \lambda, \bar{u}/u_0) - B_1\left(\underline{M}(N), b\right)/\beta(N)^2 - S_1\left(N - \underline{M}(N), s\right) \times \left[1 - \left(1/\beta(N)^2\right)\right].$$

We combine the solutions to the three cases as follows:

$$N^* = \begin{cases} N_1^* & \text{if } \underline{M}\left(N_1^*\right) \le \hat{M}\left(b,s,N_1^*\right) \le N_1^* & \text{(Case 1)} \\ N_2^* & \text{if } \hat{M}\left(b,s,N_2^*\right) > N_2^* & \text{(Case 2)} \\ N_3^* & \text{if } \hat{M}\left(b,s,N_3^*\right) < \underline{M}\left(N_3^*\right) & \text{(Case 3).} \end{cases} \quad (11)$$

4.4 Discussion of the Equilibrium

Comparative static analysis can be conducted using (11) and the implicit function theorem to determine how the model parameters affect N^* and the optimal combination of carrying brands and holding sales. The comparative static results for N^* are summarized in Table 1, and the formal derivations are provided in our working paper (Xia and Sexton 2007). Here we discuss the intuition behind some of the more important and interesting results.

The marginal revenue curve, $MR(N\,|\theta_0, \lambda, \bar{u}/u_0)$, from adding offers is downward sloping. Thus, depending upon the values of the model parameters, $MR(\cdot)$may intersect one of the three cases of the marginal cost curve, and this intersection yields the optimal number of total offers. Three parameters shift marginal revenue: λ, the extent of consumer heterogeneity; θ_0, the base level of consumer income; and $\bar{u}/u_0$, the feasible range of the quality space. Increases in each of these parameters increase marginal revenue for all three cases, in general increasing the equilibrium number of offers for each case. These results, because they affect the revenue side of the problem, would also generally be robust to alternative specifications of cost, such as an increasing cost of quality.[12]

The marginal cost curve, $MC(N\,|\cdot) = \partial TC^*/\partial N$, of adding offers depends upon which, if any, constraint of the cost-minimization problem is binding. For unconstrained case 1, the optimal distribution of N between brands and sales depends on the cost factors of both carrying brands and holding sales. Given a value of N, an increase in the cost factor of one tool of providing offers causes the seller to reduce the number of offers provided through this tool and to increase the use of the alternative tool to provide offers. Thus, for example, the effect of an increase in b, the cost of carrying brands, on $MC(N\,|\cdot)$ depends on how the marginal cost of holding sales changes with the number of sales. An increase in b shifts $MC(N\,|\cdot)$ down (up) if the marginal cost of holding sales is decreasing (increasing). The similar result applies for an increase in s.

Thus, an increase in either b or s may not cause a seller to reduce the total number of offers for the unconstrained case. A seller provides more offers when the cost of using one tool increases if there is decreasing marginal cost for using the alternative tool to provide offers.[13] Figure 2 illustrates this result. The equilibrium in panel (a) is represented by N^* total offers, M^*brands, and $H^* = N^* - M^*$sales. Panel (b) depicts an increase in b, which results in fewer brands, more sales offers, and more total offers $\left(N' > N^*\right)$, the latter result due to the decrease in the marginal cost of adding offers $\left(\partial TC'/\partial N < \partial TC^*/\partial N\right)$ caused by the decreasing marginal cost of holding sales.

We do not see this behavior in the constrained cases 2 and 3 because the constraints on the number of brands or offers in those cases restrict a seller from switching between carrying brands and holding sales in response to changes in one or both cost factors. Thus, an increase in b increases $MC(N\,|\cdot)$ and results in fewer offers for cases 2 and 3, and an increase in s increases $MC(N\,|\cdot)$ and is associated with fewer offers for case 3.[14]

Another interesting result is that a seller may switch from using only one type of instrument (multiple brands or periodic sales of a single brand) to using both instruments to induce consumer self selection if the range of the quality space becomes wider, consumers become more heterogeneous, and/or the consumers' income increases. An increase in one or more of these factors, $\bar{u}/u_0$, λ, and θ_0, increases marginal revenue, making it profitable to provide additional

[12] Some caveats apply as to the effect of $\bar{u}/u_0$. First, the effect on the equilibrium number of offers is indeterminate for the minimum-brands-plus-sales case because both MR and MC shift in that case, with the MC shift due to the possible need to carry more brands to cover a wider quality range. In a model with increasing procurement cost of product quality, if the procurement cost increases rapidly in the quality level, the seller may choose not to provide the offer with the highest quality. In this case, the width of the quality range would have no effect on the total number of offers.

[13] Even if the cost factors of both tools increase jointly, a seller may still provide more offers if the effect of the decreasing marginal cost of using one tool is strong enough to offset the effect of the increasing marginal cost of using the other tool.

[14] The directions of the effects of the brand-carrying and sales-holding cost parameters should remain the same, although the magnitudes of the effects may change, in the presence of an increasing quality cost because the seller also considers the different procurement costs when choosing the total number of offers and the optimal combination of selling instruments.

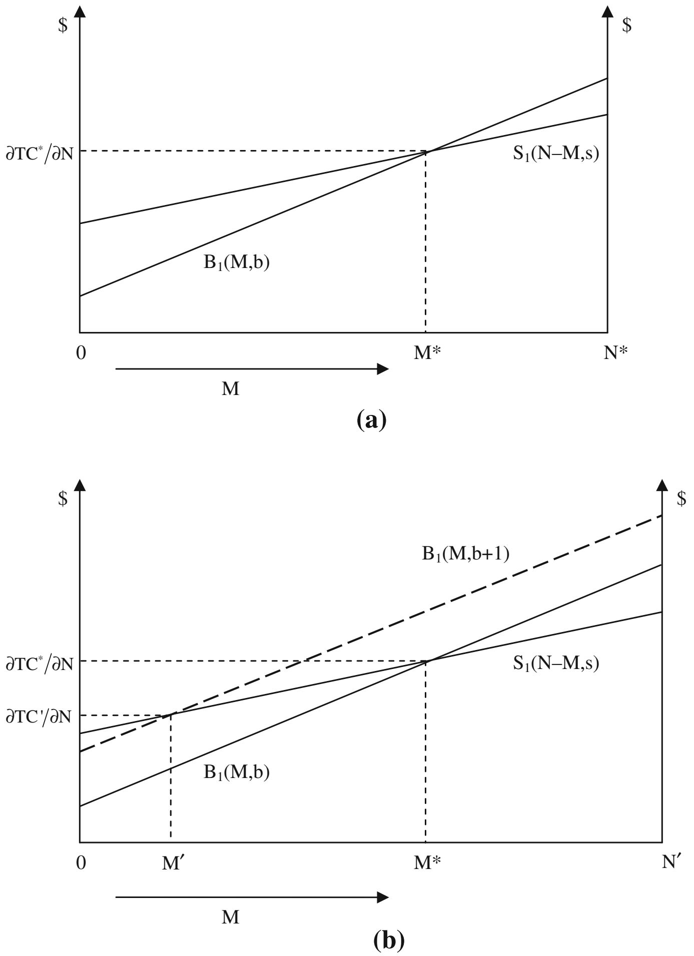

Fig. 2 The effect of the cost factor of carrying brands when there is decreasing marginal cost of holding sales. **a** The benchmark case, **b** The case when b increases to b+1

offers, which, in turn, may cause the seller to use the other instrument of price discrimination that previously was too costly to use.[15]

5 Conclusions

We have presented a model to study a monopolist's optimal choices in terms of carrying quality-differentiated brands or varieties and holding periodic sales. Given the same discounted

15 Lal and Rao (1997) established a duopoly equilibrium between retailers in which one retailer adopts a NS strategy and the other adopts a high-low pricing strategy. Our model shows that the emergence of NS and high-low retailers need not be due to competitive market positioning among them but instead can be due to the factors noted in this discussion. Our expanded working paper (Xia and Sexton 2007) provides numerical examples to illustrate these points.

price, consumers are indifferent between consuming a low-quality brand now and consuming a high-quality brand later, as long as the two alternatives provide the same present value of utility.

Thus, contemporaneous price discrimination through carrying various brands and intertemporal price discrimination through holding sales are substitute marketing strategies for a seller to induce self selection among consumers and generate revenue from them. This substitutability allows a seller to use different combinations of carrying brands and holding sales, with the specific optimal choice determined by various characteristics of the seller's store(s), products, and customers. Prior research, however, has mostly studied either tool in isolation.

Our analysis helps in understanding why different sellers of a particular product often use divergent selling strategies, and why some product categories are used frequently for sales, while others seldom, if ever, go on sale. A no-sales marketing strategy is more likely to emerge for products with a narrower quality range, relative homogeneity among potential consumers, and high sales costs relative to brand carrying costs. Conversely, product lines featured on sale will be those with more heterogeneous consumers, a wider quality range, and relatively low sales costs.

We also showed how various factors affect a seller's strategy in terms of the optimal number of offers, and the seller's choice between using only one type of instrument and using both types of instruments to conduct price discrimination. A seller may increase the total number of offers when it becomes more costly to carry brands or hold sales if there are decreasing marginal costs for the alternative selling tool. A wider product quality range, greater heterogeneity among consumers, and higher values of income of consumers all cause a seller to provide more offers and to be more likely to use both multiple brands or varieties and periodic sales to induce self selection.

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